

SPECIAL CLASSICAL CLASSES OF UNIVALENT FUNCTIONS WITH PQ-Chebyshev Polynomials

Farhat Shaheen¹, Ghulam Abbass^{*2,3}, Hisamuddin Shaikh³, Israr Ahmed⁴, Shazia Soomro⁵, Saima Mustafa⁶, Muhammad Ishfaq⁷

^{1,6}Department of Mathematics, Faculty of Science, PMAS-Arid Agriculture University Rawalpindi

^{*2,7}School of Mathematics and Statistics, Central South University, Changsha 410083, China

^{3,4}Department of Mathematics, Shah Abdul Latif University, Khairpur mirs, Sindh, Pakistan

⁵PAMH government pakistan degree college Khairpur

^{*2,3}g_abbass@yahoo.com

DOI: <https://doi.org/10.5281/zenodo.17759356>

Keywords:

Univalent function; Special classes; p, q -Chebyshev Polynomials

Article History

Received: 09 October 2025

Accepted: 18 November 2025

Published: 29 November 2025

Copyright @Author

Corresponding Author: *
Ghulam Abbass

Abstract

This article defines a new subclass of analytic univalent function with the concept of (p, q) Chebyshev polynomials and observe constants estimates, which simplify certain previously defined class. The new class, estimates for the Taylor-Maclaurin constants and the Fekete-Szego inequality for $|r_2|$ and $|r_3|$ constants bound of new subclass have been obtained. These constants estimates play a crucial role in understanding the behavior of the functions within this subclass.

1 Introduction

The scientists work on q -calculus, which includes q -number with a single base q . Chakrabarti and Jagannathan [8] independently looked into (p, q) -calculus, which contains (p, q) -number with independent variables, two variables p and q , around the same period (1991). In the mathematical literature, Wachs and White [22] developed the (p, q) number which is used to derive the production function of the joint distribution of statistics pairs, or the (p, q) Stirling number. A number of mathematical and physical issues make (p, q) calculus necessary. Since 1991, numerous mathematicians and physicists have developed the (p, q) calculus in a variety of study fields based on the previously described works. For each number n , the twin-basic number or (p, q) -number is defined as

$$[n]_{p,q} = \frac{p^n - q^n}{p - q} = p^{n-1} + p^{n-2}q + \dots + pq^{n-2} + q^{n-1}$$

This is a natural generalization of the q -number. P is a post-Quantum calculus which is the most generalized form of Q calculus.

Post Quantum Number

We define the (p, q) numbers and examine their properties, such as the addition and subtraction formulas, multiplication and division rules, and so on. However, we can only apply the addition formula. Let us give

the following definition of the (p, q) -number. The (p, q) numbers are defined as

$$[n]_{pq} = \frac{p^n - q^n}{p - q},$$

which is symmetric, that is,

$$[n]_{p,q} = [n]_{q,p}, \quad \text{see [24, 25].}$$

Application of pq calculus

Q-calculus has played a significant part in physics phenomena as a link between the realms of mathematics and physics. For example, Fock [14] used the q -difference equation to study the atomic symmetry of hydrogen. Furthermore, q -calculus has various applications in current mathematical analysis, such as combinatorics, number theory, quantum theory, physics, statistics relativity the- sis, orthogonal polynomials, basic hypergeometric functions, see also [20, 5, 7]. Both mathematicians and physicists are interested in the broad topic of frac- tional calculus. The combination of analytical function theory with fractional calculus has resulted in the development of a number of mathematical models that use fractional differential equations [3]. The q -calculus is a variant of clas- sical calculus that does not include the concept of limits, with q standing for quantum. In [4, 10], Jackson began employing the q -calculus. The first coefficients of a new class $L(\alpha, t)$ were calculated by Altinkaya in 2018. Research on a subclass of univalent functions is done in [17] using Chebyshev polynomials. The Komatu Integral operator has been applied recently to study Chebyshev polynomials and an original subclass of univalent functions [23]. Recent contri- butions to univalent function theory, including Chebyshev polynomials, can be observed in [6, 13, 18]. Let A represent the class of analytical functions of the form

$$f(u) = u + \sum_{n=2}^{\infty} a_n u^n, \quad u \in U := \{u \in C : |u| < 1\},$$

which satisfies the normalization conditions $f(0) = 0, f'(0) = 1$. The author of [9], constructed the following significant univalence condition for the subclass S using the inclusion relation.

Theorem (1.1)

Let $f \in A$ satisfy

$$\Re \left\{ \frac{2uD_{pq}f(u) + u^2D_{pq}^2f(u)}{f(u) + uD_{pq}f(u)} \right\} > 0 \tag{1}$$

Then $f(u)$ is satarlike univalent in U .

Example 1.

$$f(u) = \frac{u}{(u-1)^2}$$

is the Koebe function that is starlike in the open unit disk. (1932, Marx [16]) Geometric function theory has a long history of the Fekete-Szego functional $|r_3 - \phi r_2^2|$. In 1933, Littlewood's theory Unity defines the boundaries of func- tions, which was refuted by the authors in [19]. Particularly in subcategories within the univalent function family, the functional has drawn a lot of atten- tion. According to some research, this issue has drawn attention from academics recently (see, for instance, [12]).

Definition 1.1 Chakrabarti and Jagannathan [15]. The (p,q) -differential of a function f is defined as,

$$d_{p,q}f(x) = f(p_x) - f(q_x)$$

Similar to the q -differential in [1], a contrast of the (p, q) -differential from the typical one is the absence of balance in the differential of the result of two functions. There are no investigations of q -Chebyshev polynomials related to univalent functions in the literature that we are aware of. The main goal of this research was to examine the characteristics of univalent functions related to q -Chebyshev polynomials. In this paper, preliminary coefficient calculations for the Fekete-Szego problem for univalent function subclasses $HB(p, q, n, a)$ are computed using the $4pq$ -Chebyshev polynomial expansion. The authors focused on the bounds of factor functions for novel subclasses of univalent functions using certain pq -Chebyshev polynomials.

2 Bounds of the constants and Fekete-Szego In- equalities

A function $f \in A$ is considered to be in the class,

$HB(p, q, n, a)$, where $\frac{1}{2} < a < 1, 0 < q < p < 1, u \in C, -1 \leq n \leq 2$, and $u \in U$

$$\frac{2uD_{pq}f(u) + u^2D_{pq}^2f(u)}{f(u) + uD_{pq}f(u)} \prec \omega(p, q, n, a) \tag{2}$$

where D_{pq} is the pq -differential operators and \prec is the sign for subordination [21].

$$H_v(p, q, n, a) = \sum_{n=0}^{\infty} H_v(p, q, n, a)u^n, \left(\frac{1}{2} < a < 1, 0 < q < p < 1, u \in C, -1 \leq n \leq 2\right)$$

Where

$$H_v(p, q, n, a) = \sum_{k=0}^{\frac{n}{2}} (pq)^{k^2} \binom{n-k}{k}_{pq} \frac{(p,-q);(p,q)}{(p,-q);(p,q)} t^k s^{\rho-2k} \tag{3}$$

are called pq -Chebyshev polynomials of the second kind.

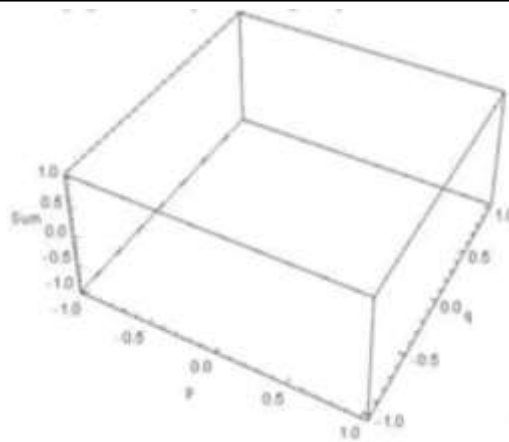


Figure 1: The figure represents the Chebyshev polynomials of second kind. The value of this polynomial lies in $[-1, 1]$ and $n \geq 2, p < 1, q \leq 1$.

we have

$$H_v(p, q, n, a) = (p^k + q^q a)V_{k-1}(p, q, n, a) + (pq)^{k-1}V_{k-2}(p, q, n, a) \tag{4}$$

Using these identities in main results:

$$H_1(p, q, n, a) = a + apq$$

$$H_2(p, q, n, a) = npq + a^2 + a^2 p^2 q^2 + pq a^2 + a^2 p^3 q^3$$

$$H_3(p, q, n, a) = pqna + p^3 q^3 na + np^2 q^2 a + anq^4 p^4 + a^3 (q^6 p^6 + p^5 q^5 + q^4 + 2p^3 q^3 + p^2 q^2 + pq + 1)$$

Remark 1. We can see that

$$H_v(1, -1, a) = H_v(a)$$

where $H_v(a)$ is the classical Chebyshev polynomials of the second kind.

The evaluations on the Maclaurin series factors $|r_2|$ and $|r_3|$ for functions in the class $HB(p, q, n, a)$ are resolved. Diaz and Osler [2] used q -calculus fractional differential operator to define analytical function in U . The purpose of this study is to propose new notions of (p, q) differential. We look at the fractional (p, q) -difference operators of the Chebyshev polynomials of the second class. A study of this fractional (p, q) -calculus is expected to be very important in the development of the (p, q) -function theory.

Main Results Theorem 2.1

Let $f(u) \in HB(p, q, n, a)$. Then

Let $f(u) \in HB(p, q, n, a)$. Then

$$|r_2| \leq \frac{2a(1 + pq)}{2p + 2q - 1}$$

for all $\phi \in \mathbb{R}$

$$|r_3| \leq \frac{6a^2 p + 6a^2 q - 3a^2 + 6a^2 p^2 q + 6a^2 p q^2 - 6a^2 p q + 4a^2 + 4a^2 p^3 q^2 + 4a^2 q + 4a^2 p^2 q^3 - 2a^2 - 2a^2 p^2 q^2}{(1 + pq)^2 (2p + 2q - 1)} +$$

$$\frac{2a + 2aq + 2nq}{p^3 + q^3 + p^2 + q^2 + 2p^2 q + 2p q^2 + pq - 1}$$

$$|r_3 - \phi r_2^2| \leq \begin{cases} \frac{2a}{(1+pq)^2} \\ \forall \phi \in [\phi_1, \phi_2] \end{cases}$$

$$|r_3 - \phi r_2^2| \leq \left\{ \frac{2a}{(1+pq)^2} \left| \frac{a^2(1+pq)(1+p^2q^2) + pqn}{(1+pq)a} + \frac{(p+q+1)(1+pq)a}{2p+2q-1} - \frac{2\phi a(p^3+q^3+p^2+q^2+2p^2q+2pq^2+pq-1)(1+pq)}{(2p+2q-1)^2} \right| \leq 1 \right\}$$

for all $\phi \in [\phi_1, \phi_2]$ where
where

$$\phi_1 = \frac{\alpha + \beta + \nu}{\eta + \mu} \tag{5}$$

$$\begin{aligned} \alpha &= 2a^2 p^2 + a^2 p^3 q^4 + 4a^2 p^4 q + 2a^2 p^3 q^4 + 3a^2 p + 4a^2 p^2 q^2 + a^2 pq \\ \beta &= -4pqn - 4p^2qn - 4pq^2 - a - a^2 p^3 q^3 - 4pq^3n + pqn + 8p^2q^2n \\ \nu &= -8apq + 4aqp - 4ap^3q - 4apq^3 - apq + 8ap^2q^2 + 4apq^2 + 4apq^2 \\ \eta &= 4a^2 p^5 q^2 + 2a^2 p^2 q^5 + 2a^2 p^3 + 2a^2 q^3 + 4a^2 p^2 q + 4a^2 p^3 q^2 \\ \mu &= 2a^2 p^2 q^4 + 4a^2 p^4 q^3 + 4a^2 p^3 q^4 + 2a^2 p^3 q^3 + 2a^2 p^2 q^2 + 4a^2 p^4 q^4 + 4a^2 p^3 q \end{aligned}$$

$$\phi_2 = \frac{\alpha + \beta + \nu}{\eta + \mu} \tag{6}$$

$$\begin{aligned} \alpha &= 2a^2 p + 3a^2 p^3 q^2 + 4a^2 p^2 q^4 + 2a^2 p^4 q^4 + 3a^2 p^3 q^3 + 4a^2 pq^2 + apq \\ \beta &= -a^3 p^3 q^3 - 4pq^3n - 4p^3qn - 8p^2q^2n - 4p^2n + 4a^2 p^2 + 4a^2 q^2 + a \\ \nu &= 8apq + 4aqp + 4apq^3 + 4apq^2 - apq + 8ap^2q + 4apq^2 \\ \eta &= 4a^2 p^5 q^2 + 2a^2 p^2 q^5 + 2a^2 p^3 + 2a^2 q^3 + 4a^2 p^2 q^4 + 4a^2 p^2 q^2 + 4a^2 p^2 q \\ \mu &= 2a^2 p^2 q^4 + 4a^2 p^3 q^3 + 4a^2 p^3 + 4a^2 p^2 q^2 + 2a^2 p^4 q^4 + 4a^2 p^2 q^3 + 8a^2 p^3 q^3 \end{aligned}$$

Where $\phi_1 \leq \phi \leq \phi_2$ and $\phi_1, \phi_2 \in \mathbb{R}$

Proof. Let $f(u) \in HB(p, q, n, a)$. From (3), the results are,

$$\frac{2uD_{pq}f(u) + u^2D_{pq}^2f(u)}{f(u) + uD_{pq}f(u)} = 1 + H_1(p, q, n, a)\omega(u) + H_2(p, q, n, a)\omega^2(u) + H_3(p, q, n, a)\omega^3(u) + \dots \tag{7}$$

for some analytical function ω such that $\omega(0) = 0$ and $|\omega| < 1$ for all $u \in U$. Taking it from there, (8) we get

$$\frac{2uD_{pq}f(u) + u^2D_{pq}^2f(u)}{f(u) + uD_{pq}f(u)} = 1 + H_1(p, q, n, a)k_1u + [H_2(p, q, n, a)k_2u + H_2(p, q, n, a)k_2^1u]u^2 + \dots \tag{8}$$

$$|\omega(u)| = |k_1u + k_2u^2 + k_3u^3 + \dots| < 1 \text{ and } u \in U$$

$$|k_v| \leq 1, v \in \mathbb{N} \tag{9}$$

$$|k_2 - \zeta k_2^1| \leq \max(1 + |\zeta|), \forall \zeta \in \mathbb{R} \tag{10}$$

As the result of (9), it follows that

$$\frac{2p + 2q - 1}{2} r_2 = H_1(p, q, n, a) k_1 \tag{11}$$

$$\frac{(p^3 + q^3 + p^2 + 2p^2q + 2pq^2 + pq - 1)r_3}{2} - \frac{(2p^2 + 2q^2 + 4pq + p + q - 1)r_2^2}{4}$$

$$H_1(p, q, n, a)k_2 + H_2(p, q, n, a)k_1^2 \tag{12}$$

From identities 1, 2 and 12, we get

$$|r_2| \leq \frac{2a(1 + pq)}{2p + 2q - 1} \tag{13}$$

Then, using (12) in (13), we can find the bound on $|r_3|$.

$$|r_3| = \frac{2H_1(p, q, n, a)k_2}{p^3 + q^3 + p^2 + 2p^2q + 2pq^2 + pq - 1} + \frac{H_2(p, q, n, a)k_1^2}{p^3 + q^3 + p^2 + 2p^2q + 2pq^2 + pq - 1} +$$

$$\frac{(4p^2 + 4q^2 + 8pq + 2p + 2q - 2)H_1(p, q, n, a)k_1^2}{(p^3 + q^3 + p^2 + 2p^2q + 2pq^2 + pq - 1)(2p - 2q - 1)^2} \tag{14}$$

In light of identities and(10), we have

$$|r_3| \leq \frac{6a^2p + 6a^2q - 3a^2 + 6a^2p^2q + 6a^2pq^2 + 4a^2 + 4a^2p^3q^2 + 4a^2p^2q^3 - 2a^2 - 2a^2p^2q^2}{(1 + pq)^2(2p + 2q - 1)} +$$

$$\frac{2a + 2aq + 2nq}{p^3 + q^3 + p^2 + 2p^2q + 2pq^2 + pq - 1} \tag{15}$$

From (15) and (11), for $\phi \in \mathbb{R}$ we get

$$|r_3 - \phi r_2^2| = \frac{2H_1(p, q, n, a)}{p^3 + q^3 + p^2 + 2p^2q + 2pq^2 + pq - 1} \left| k_2 + \left\{ \frac{H_2(p, q, n, a)}{H_1(p, q, n, a)} + \frac{p + q + 1}{2p + 2q - 1} H_1(p, q, n, a) \right\} \right| +$$

$$\left| -2\phi \frac{p^3 + q^3 + p^2 + 2p^2q + 2pq^2 + pq - 1}{(2p + 2q - 1)^2} \right| \tag{16}$$

Based on (11), we arrive at the following conclusion

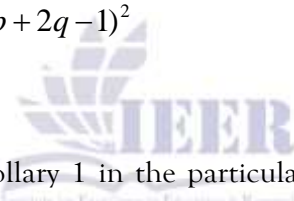
$$|r_3 - \phi r_2^2| \leq \frac{2H_1(p, q, n, a)}{p^3 + q^3 + p^2 + 2p^2q + 2pq^2 + pq - 1} \max \left\{ 1, |k_2 + \frac{H_2(p, q, n, a)}{H_1(p, q, n, a)} + \frac{p+q+1}{2p+2q-1} H_1 - \right. \\ \left. 2\phi \frac{p^3 + q^3 + p^2 + 2p^2q + 2pq^2 + pq - 1}{(2p+2q-1)^2} H_1(p, q, n, a) k_1^2 \right\} \quad (17)$$

We have achieved this by using (identities) in (18).

$$|r_3 - \phi r_2^2| \leq \frac{2a}{(1+pq)^2} \max \left\{ 1, \frac{a^2(1+pq)(p^2+q^2)+pqn}{a(1+pq)} + \frac{(p+q+1)(1+pq)a}{2p+2q-1} \right\} - \frac{2a\phi(p^3+q^3+p^2+2p^2q+2pq^2+pq-1)(1+pq)}{(2p+2q-1)^2} \\ 2\phi \frac{p^3+q^3+p^2+2p^2q+2pq^2+pq-1}{(2p+2q-1)^2} \quad (18)$$

Because a is greater than zero, we have

$$|r_3 - \phi r_2^2| \leq \frac{2a}{(1+pq)^2} \left| \left(\frac{a^2(1+pq)(p^2+q^2)+pqn}{a(1+pq)} + \frac{(p+q+1)(1+pq)a}{2p+2q-1} \right) \right| \\ \left| - \frac{2a\phi(p^3+q^3+p^2+2p^2q+2pq^2+pq-1)(1+pq)}{(2p+2q-1)^2} \right| \leq 1 \quad (19)$$



Theorem 2.1. This brings us to Corollary 1 in the particular case where $q = 1$, $p = 1$, and $n = -1$.

Corollary 1. Let $f(u) \in HB(a)$. Then

$$|r_2| \leq \frac{4a}{3}$$

$$|r_3| \leq \frac{31a^2 + 6a - 2}{12} \quad \forall \phi \in [\phi_1, \phi_2]$$

$$|r_3 - \phi r_2^2| \leq \left\{ \frac{a}{2} \left| \frac{4a^2 - 1}{2a} + 3a - \frac{32}{4} \phi a \right| \right\}$$

for all $\phi \in [\phi_1, \phi_2]$

$$\phi_1 = \frac{24a^2 + 18a - 9}{66a^2}$$

$$\phi_2 = \frac{24a^2 - 18a - 9}{66a^2}$$

Corollary 2.

Theorem 2.1, This brings us to Corollary 2 in particular case when $p = 1$.

$$|r_2| \leq \frac{2a(1+q)}{1+2q}$$

And

$$|r_2| \leq \frac{2a^2(2q^3 + 2q^2 + 5q + 1)}{(q+1)^2(1+2q)} + \frac{2a + 2aq + 2qn}{q^3 + 3q^2 + 3q + 1}$$

and for $\phi \in \mathfrak{R}$

$$|r_3 - \phi r_2^2| \leq \frac{2a}{(1+q)^2}$$

for all $\phi \in [\phi_1, \phi_2]$

$$|r_3 - \phi r_2^2| \leq \frac{2a}{(1+q)^2} \left| \frac{a^2(1+q)(1+q^2) + qn}{(1+q)a} + \frac{(2+q)(1+q)a}{1+2q} - \frac{2\phi a(q^3 + 3q^2 + 6q + 1)(1+q)}{(1+2q)^2} \right| \leq 1$$

for all ϕ_1, ϕ_2 where

$$\phi_1 = \frac{3a^2 + 4a^2q^3 + 7a^2q^2 + 8a^2q + 2a^2q^4 - a - 5aq + qn - 8aq^2 + 4nq^2 - 4aq^3 + 4nq^3}{2a^2q^5 + 10a^2q^4 + 20a^2q^3 + 20a^2q^2 + 10a^2q + 2a^2}$$

$$\phi_2 = \frac{3a^2 + 4a^2q^3 + 7a^2q^2 + 8a^2q + 2a^2q^4 + a + 5aq + qn + 8aq^2 + 4nq^2 + 4aq^3 + 4nq^3}{2a^2q^5 + 10a^2q^4 + 20a^2q^3 + 20a^2q^2 + 10a^2q + 2a^2} \quad (20)$$

Conflict of interest

The authors declare no conflict of interest.

Authors Contributions

All authors have equally Contributed.

Data availability

No data is used for this study.

3 Conclusion

The field of pq-calculus has emerged as a fascinating area of study, with promising implications for various mathematical disciplines. In particular, it is pq-calculus expected to be very important in the development of the pq-function theory, analytical number theory, special polynomials. In the previous work the author Altnkaya and Yalcin [11] has established analytical univalent class $HB(q, n, a)$ linked to the q-Chebyshe polynomial. In this paper we have investigated Fekete-Szego problem and constants estimates $|r_2|$ and $|r_3|$ of the above mentioned

class $HB(p,q,n,a)$. In this particular work we have obtained generalized form of q-calculus and have obtained this class by using pq-operator.

References

Isra Al-shbeil, Jianhua Gong, and Timilehin Gideon Shaba. Coefficients inequalities for the bi-univalent functions related to q-babalola convolution operator. *Fractal and Fractional*, 7(2):155, 2023.

- S Altinkaya and S Yalçın. On the chebyshev polynomial bounds for classes of univalent functions, *khayyam j. Math*, 2(1):1-5, 2016.
- S. AHSENE ALTINKAYA and SİBEL YALÇIN TOKGÖZ. On the chebyshev coefficients for a general subclass of univalent functions. *Turkish journal of mathematics*, 42(6):2885-2890, 2018.
- S. ahsene Altinkaya and Sibel Yalçın. Chebyshev polynomial bounds for a certain subclass of univalent functions defined by komatu integral operator. *Afrika Matematika*, 30(3):563-570, 2019.
- Suphawat Asawasamrit, Chayapat Sudprasert, Sotiris K Ntouyas, and Jessada Tariboon. Some results on quantum hahn integral inequalities. *Journal of Inequalities and Applications*, 2019(1):154, 2019.
- Kunle Oladeji Babalola. On some starlike mappings involving certain convolution operators. *arXiv preprint arXiv:1004.2602*, 2010.
- Gaspard Bangerezako. Variational calculus on q -nonuniform lattices. *Journal of mathematical analysis and applications*, 306(1):161-179, 2005.
- R Chakrabarti and R Jagannathan. A (p, q) -oscillator realization of two-parameter quantum algebras. *Journal of Physics A: Mathematical and General*, 24(13):L711, 1991.
- R Chakrabarti and R Jagannathan. A (p, q) -oscillator realization of two-parameter quantum algebras. *Journal of Physics A: Mathematical and General*, 24(13):L711, 1991.
- K Dhanalakshmi, D Kavitha, and A Anbukkarasi. Coefficient estimates for bi-univalent functions in connection with (p, q) chebyshev polynomial. *J. Math. Comput. Sci.*, 11(6):8422-8429, 2021.
- JB Diaz and Thomas J Osler. Differences of fractional order. *Mathematics of Computation*, 28(125):185-202, 1974.
- PL Duren. Grundlehren der mathematischen wissenschaften. *Univalent Functions*; Springer: New York, NY, USA; Berlin/Heidelberg, Germany, 259, 1983.
- M Fekete and G Szegő. Eine bemerkung über ungerade schlichte funktionen. *Journal of the london mathematical society*, 1(2):85-89, 1933.
- Vladimir Fock. Zur theorie des wasserstoffatoms. *Zeitschrift für Physik*, 98(3):145-154, 1935.
- FH Jackson, Tanaka Fukuda, Ogilvie Dunn, and English Majors. On q -definite integrals. *Quart. J. Pure Appl. Math*, 41(15):193-203, 1910.
- Victor Kac and Pokman Cheung. Classical partition function and euler's product formula. In *Quantum Calculus*, pages 37-42. Springer, 2002.
- Muhammet Kamali, MURAT ÇAGLAR, Erhan Deniz, and Mirzaolim Turabaev. Fekete-szegő problem for a new subclass of analytic functions satisfying subordinate condition associated with chebyshev polynomials. *Turkish Journal of Mathematics*, 45(3):1195-1208, 2021.
- N Magesh and J Yamini. Fekete-szegő problem and second hankel determinant for a class of bi-univalent functions. *arXiv preprint arXiv:1508.07462*, 2015.
- Arnold Marx. Untersuchungen über schlichte abbildungen. *Mathematische Annalen*, 107(1):40-67, 1933.
- Pheak Neang, Kamsing Nonlaopon, Jessada Tariboon, Sotiris K Ntouyas, and Praveen Agarwal. Some trapezoid and midpoint type inequalities via fractional (p, q) -calculus. *Advances in Difference Equations*, 2021(1):333, 2021.
- Timilehin Gideon Shaba, Serkan Araci, Jong-Suk Ro, Fairouz Tchier, Babatunde Olufemi Adebesein, and Saira Zainab. Coefficient inequalities of q -bi-univalent mappings associated with q -hyperbolic tangent function. *Fractal and Fractional*, 7(9):675, 2023.

SR SWAMY, DANIEL BREAZ, LUMINITA-IOANA COTÎRLĂ, and KALA VENUGOPAL. Bi-univalent function subclasses with (p, q) -derivative operator linked to horadam polynomials. *Kragujevac Journal of Mathematics*, 50(8):1279-1296, 2026.

Eszter Szatmari and Sahsene Altinkaya. Coefficient estimates and feketé-szegő inequality for a class of analytic functions satisfying subordinate condition associated with chebyshev polynomials. *Acta Univ. Sapientiae Math*, 11(2):430-436, 2019.

Michelle Wachs and Dennis White. p, q -stirling numbers and set partition statistics. *Journal of Combinatorial Theory, Series A*, 56(1):27-46, 1991.

Michelle Wachs and Dennis White. p, q -stirling numbers and set partition statistics. *Journal of Combinatorial Theory, Series A*, 56(1):27-46, 1991.

