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MATHEMATICAL MODELING OF KNOWLEDGE TRANSMISSION IN A CLASSROOM USING THE SIR FRAMEWORK

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Abstract
This study presents a mathematical analysis of a Susceptible–Informed–Recovered (SIR) model to describe the spread of knowledge in a classroom environment. The model divides students into three compartments: susceptible students who have not yet understood a concept, informed students who have learned the concept and can assist others, and recovered students who have fully mastered the concept. The model incorporates learning transmission and knowledge mastery rates to represent realistic classroom interactions. The existence and uniqueness of solutions are established using the Lipschitz condition, ensuring that the model is mathematically well-posed. Positivity of solutions is proved to guarantee that all state variables remain non-negative over time. An invariant region is derived to demonstrate that the total number of students remains bounded. The knowledge-free equilibrium is determined and analyzed as the baseline state where no students have learned the concept. A threshold parameter analogous to the basic reproduction number $(\mathcal{R}_0)$ *is computed to measure the effectiveness of knowledge transmission in the classroom. Local stability of the equilibrium is examined through Jacobian matrix and eigenvalue analysis, showing stability when the threshold parameter is less than unity. Furthermore, a Lyapunov function is constructed to establish global stability under the same condition. The results provide theoretical insights into how knowledge spreads through peer interaction and instruction. Overall, the model offers a useful mathematical framework for understanding learning dynamics and improving teaching strategies in educational settings.*

1 Introduction

Mathematical modeling is an effective scientific method to describe the real-life phenomena by means of mathematical equations and logical dependencies [1,2]. It enables researchers to reduce the complexity of systems and investigate their behavior in terms of a quantitative approach. Mathematical models find wide application in different fields like biology, economics, engineering, epidemiology and social sciences. Mathematical modeling is done to transform real-life problems into mathematical terms that can be analyzed and interpreted in systematical ways. The modeling involves making assumptions about the system and introducing variables to explain how various constituents of the system interact.

Mathematical modeling assists the researchers in knowing patterns, relationships, and trends of a particular process. It also permits the scientists to model various scenarios and forecast the potential results without conducting expensive experiments in the real world. Mathematical models give insights in a number of situations which are not easily observable. Moreover, the models assist the decision makers in developing effective policies and strategies. Over the past years, mathematical modeling has been used in the education field to learn the dynamics of learning, teaching, and knowledge transfer amongst students. Through mathematical methods, scientists are able to explore the possibility of how ideas and knowledge are disseminated in a group of people. As such, mathematical modeling offers an organized system of studying complicated processes as well as enhancing the knowledge of dynamic systems [3].

One of the most famous compartmental models to explain the process of spreading infectious diseases in the population is the Susceptible, Infected and Recovered (SIR) model [4, 5]. Through this model, the entire population is classified into three groups which include; the susceptible population, the infected population, and the recovered population. The vulnerable are those who seek to be infected, the infected are those who are already infected and transmit the infection and the recovered are those who have already survived and developed immunity. SIR model involves the movement of individuals between compartments with the help of a set of differential equations [6]. It is through the contact between the infected and the susceptible persons that the disease is spread. The recovery process transfers the people in the infected category to the recovered category. This model has found immense application in epidemiology both in understanding how diseases are transmitted and in prediction of an outbreak [7]. The SIR model is used to examine how various factors like the rate of spreading the diseases because of the transmission rates, the recovery rates and so on have an influence on the disease spread. The simplicity and versatility of the SIR framework are appropriate to study a wide variety of dynamic processes not only in the field of epidemiology. Consequently, SIR model has also been modified to investigate the distribution of information, behavior and ideas in social systems. Dissemination of knowledge in a classroom may be considered a dynamic process that occurs in the same way as that of information dissemination in a social network. Within a school setting, students and their teacher develop interactions, and as a result, knowledge can be exchanged between students and the teacher. There are those students, who may have no knowledge on a specific concept, and others who may already know the concept and assist in clarifying it to their companions. Knowledge is diffused amongst students through discussions, group work as well as classroom activities. Gradually, additional students become conversant with the concept and thus end up attaining mastery. A number of factors can affect this learning process; some of them are peer interaction, quality of teaching, student motivation, and independent study. Knowing the knowledge dissemination in a classroom would help an educator to develop better teaching methods as well as to increase student performance. The analysis of this process is systematically analyzed through mathematical modeling. Researchers can examine how student interactions can lead to the spread of knowledge by acting on behalf of students at various learning stages. Thus, the examination of knowledge dissemination in a classroom is one of the essential moves towards enhancing the efficiency of the educational systems and facilitating the collaborative learning conditions [8].

The SIR model can be adjusted to the transmission of knowledge in a classroom by explaining its subunits in a school setting. In this context, vulnerable individuals are those students yet to comprehend a given concept. The infected are the students who are familiar with the concept and contribute to spreading the knowledge among others. Recovered people are those students who have mastered the concept well and do not require help anymore. The process of exchanging knowledge among the susceptible and informed students relates to the process of the spread of infections within the epidemiological models.

With the help of the differential equations, a researcher can arrange the rate of knowledge propagation in the classroom, and the effect of various variables on the learning process. This method lets the educators estimate the efficiency of pedagogical methods and collaborative learning approaches. In addition, other parameters that can be included in the model are teacher influence, self-learning capability and forgetting rates. These aspects render the model more realistic and are appropriate in describing educational backgrounds. Thus, SIR framework can be a beneficial mathematical knowledge to consider the process of knowledge diffusion and enhance the learning process in classrooms.

This paper, based on the SIR modeling framework, constructs and solves a mathematical model of knowledge dissemination in a classroom. The primary aim of the given work is to research the knowledge spreading amongst students. The proposed model will separate the student population into three compartments that indicate different levels of learning. In order to examine the behavior of the model, there will be a number of theoretical analyses that will be conducted. To make sure that the model is mathematically well-defined, the first will be the existence and uniqueness of the solution. The solution will then be made to be positive to ensure that the variables of the population are not negative over time. The model will also be determined to have an invariant region to guarantee that the solutions do not go out of range in a viable biological or schooling field. Moreover, the path of the model to the knowledge-free equilibrium point will be acquired and discussed. The reproduction number will be calculated in basic form in order to measure the threshold condition of the spread of knowledge in the classroom. Lastly, the knowledge-free equilibrium in the local and global stability analysis will be conducted to get a sense of the long-term model behavior. Those analyses will give significant information about knowledge transmission dynamics and effectiveness of various educational strategies.

2 Methodology

2.1 Model Formulation

The spread of knowledge in a classroom can be modeled using the classical Susceptible–Informed–Recovered (SIR) framework, adapted from epidemiology. In this educational context, the compartments represent different stages of learning among students.

Table 1: State Variables of the Classroom Knowledge Model (1)

Variable	Description	Interpretation
$S(t)$	Susceptible students	Students who have not yet learned the concept
$I(t)$	Informed students	Students who understand the concept and help others
$R(t)$	Recovered students	Students who have fully mastered the concept

Table 2: Parameters of the Classroom Knowledge Model (1)

Parameter	Description	Interpretation
β	Peer learning rate	Rate at which students learn from their peers
γ	Mastery rate	Rate at which informed students achieve full understanding
τ	Teacher effectiveness	Rate at which the teacher directly teaches susceptible students
δ	Self-learning rate	Rate at which students learn independently
μ	Forgetting rate	Rate at which recovered students forget and become susceptible
N	Total students	Total number of students in the classroom ($N = S + I + R$)

The dynamics of knowledge transmission are described by the following system of differential equations:

$$\begin{aligned} \frac{dS}{dt} &= -\beta \frac{SI}{N} - \tau S - \delta S + \mu R, \\ \frac{dI}{dt} &= \beta \frac{SI}{N} + \tau S + \delta S - \gamma I, \\ \frac{dR}{dt} &= \gamma I - \mu R. \end{aligned} \tag{1}$$

With initial conditions

$$S(0) = S_0 \geq 0, \quad I(0) = I_0 \geq 0, \quad R(0) = R_0 \geq 0.$$

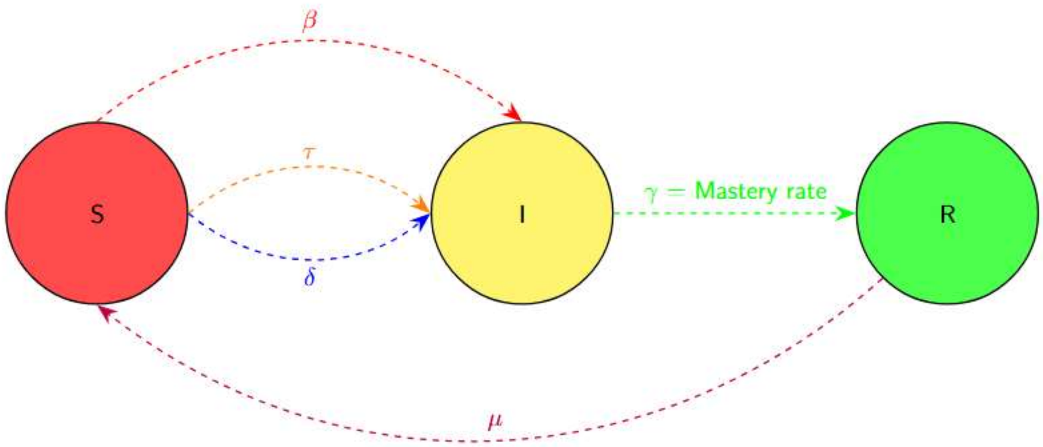

Figure 1: Classroom Knowledge Transmission SIR Model (1)

Here, the first equation represents the decrease in susceptible students due to peer interaction, teacher instruction, and self-learning, while accounting for forgetting. The second equation models the increase in informed students and their decrease as they achieve mastery. The third equation captures the accumulation of recovered students and the loss due to forgetting. This model provides a structured framework to analyze the dynamics of knowledge spread, and the effects of peer learning, teacher influence, and self-learning within a classroom setting.

2.2 Existence and Uniqueness of Solution

Define the vector function for the model (1):

$$\mathbf{X}(t) = \begin{bmatrix} S(t) \\ I(t) \\ R(t) \end{bmatrix}, \quad \mathbf{F}(\mathbf{X}) = \begin{bmatrix} f_1(S,I,R) \\ f_2(S,I,R) \\ f_3(S,I,R) \end{bmatrix},$$

where

$$\begin{aligned} f_1(S,I,R) &= -\beta \frac{SI}{N} - (\tau + \delta)S + \mu R, \\ f_2(S,I,R) &= \beta \frac{SI}{N} + (\tau + \delta)S - \gamma I, \end{aligned} \tag{2}$$

$$f_3(S,I,R) = \gamma I - \mu R.$$

The partial derivatives of f_i with respect to S, I, R are

$$\begin{aligned}
\frac{\partial f_1}{\partial S} &= -\beta\frac{I}{N} - (\tau+\delta), & \frac{\partial f_1}{\partial I} &= -\beta\frac{S}{N}, & \frac{\partial f_1}{\partial R} &= \mu, \\
\frac{\partial f_2}{\partial S} &= \beta\frac{I}{N} + (\tau+\delta), & \frac{\partial f_2}{\partial I} &= \beta\frac{S}{N} - \gamma, & \frac{\partial f_2}{\partial R} &= 0, \\
\frac{\partial f_3}{\partial S} &= 0, & \frac{\partial f_3}{\partial I} &= \gamma, & \frac{\partial f_3}{\partial R} &= -\mu.
\end{aligned} \tag{3}$$

These derivatives are continuous in $S, I, R \geq 0$ and bounded for physically meaningful population values ($0 \leq S, I, R \leq N$). Since all partial derivatives are continuous and bounded in a domain $D = \{(S,I,R) \mid 0 \leq S, I, R \leq N\}$, the function $\mathbf{F}(\mathbf{X})$ satisfies the Lipschitz condition [9]:

$$\| \mathbf{F}(\mathbf{X}_1) - \mathbf{F}(\mathbf{X}_2) \| \leq L \| \mathbf{X}_1 - \mathbf{X}_2 \|, \quad \forall \mathbf{X}_1, \mathbf{X}_2 \in D, \tag{4}$$

for some constant $L > 0$.

By the Picard–Lindelöf theorem, if $\mathbf{F}(\mathbf{X})$ is continuous and Lipschitz, then for any initial condition $\mathbf{X}(0) = \mathbf{X}_0 \in D$, there exists a unique solution $\mathbf{X}(t)$ defined at least on some interval $t \in [0, T]$. Therefore, the SIR classroom knowledge model admits a unique solution $(S(t), I(t), R(t))$ for the given initial conditions, and the solution depends continuously on the initial values.

2.3 Positivity of the Susceptible Population

Using the first equation of the SIR model (1):

$$\frac{dS}{dt} = -\beta\frac{SI}{N} - \tau S - \delta S + \mu R, \tag{5}$$

with initial condition $S(0) = S_0 \geq 0$.

Since all population compartments satisfy

$$0 \leq S(t), I(t), R(t) \leq N,$$

we can bound the negative terms and write a differential inequality:

$$\frac{dS}{dt} \geq -(\beta + \tau + \delta)S. \tag{6}$$

The solution of the linear differential equation

$$\frac{dS}{dt} = -(\beta + \tau + \delta)S, \quad S(0) = S_0,$$

is

$$S(t) = S_0 e^{-(\beta+\tau+\delta)t} \geq 0, \quad \forall t \geq 0. \tag{7}$$

$$S(t) \geq 0 \quad \text{for all } t \geq 0. \tag{8}$$

Thus, the susceptible students remains non-negative for all time [10]. Similarly, the positivity of the other compartments $I(t)$ and $R(t)$ can be proved using the same method.

2.4 Invariant Region of the SIR Model

Total number of students in the class is written as:

$$N(t) \quad = S(t) + I(t) + R(t). \tag{9}$$

Adding the three differential equation of the model (1):

$$\frac{dN}{dt} \quad = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} \tag{10}$$

$$\frac{dN}{dt} \quad = \left(-\beta\frac{SI}{N} - \tau S - \delta S + \mu R\right) + \left(\beta\frac{SI}{N} + \tau S + \delta S - \gamma I\right) + (\gamma I - \mu R) = 0. \tag{11}$$

Since $\frac{dN}{dt} = 0$, the total number of students in the class remains constant:

$$N(t) = \quad S(t) + I(t) + R(t) = S(0) + I(0) + R(0) = N, \quad \forall t \geq 0. \tag{12}$$

From the positivity proof of each compartment:

$$S(t) \geq 0, \quad I(t) \geq 0, \quad R(t) \geq 0,$$

and the total number of students in the class is constant, it follows that

$$0 \leq S(t), I(t), R(t) \leq N. \tag{13}$$

Thus, the SIR classroom knowledge model is well-posed in the invariant region [9]:

$$\Omega = \{(S, I, R) \in \mathbb{R}^3_{\geq 0} : S + I + R = N\}. \tag{14}$$

All solutions starting in Ω remain in Ω for all $t \geq 0$.

2.5 Knowledge-Free Equilibrium (KFE)

The knowledge-free equilibrium represents the state where no student has learned the concept [11]: $I^* = 0$.

At equilibrium, all derivatives are zero:

$$\frac{dS}{dt} = 0, \quad \frac{dI}{dt} = 0, \quad \frac{dR}{dt} = 0. \tag{15}$$

From the third equation of the model (1):

$$0 = \frac{dR}{dt} = \gamma I - \mu R = 0\ R^* = 0. \quad (16)$$

From the second equation of the model (1):

$$0 = \frac{dI}{dt} = \beta \frac{SI}{N} + \tau S + \delta S - \gamma I. \quad (17)$$

Since $I^* = 0$, this reduces to:

$$(\tau + \delta)S^* = 0S^* = 0 \quad \text{or} \quad \tau = \delta = 0. \quad (18)$$

In the classical SIR case without recruitment ($\tau = \delta = 0$), we have:

$$S^* = N, \quad I^* = 0, \quad R^* = 0. \quad (19)$$

Hence, the knowledge-free equilibrium is

$$E_0 = (S^*, I^*, R^*) = (N, 0,0). \quad (20)$$

2.6 Basic Reproduction Number $\mathcal{R}_0$

The basic reproduction number $\mathcal{R}_0$ measures the average number of students that an informed student can teach before mastering the concept, indicating whether knowledge will spread in the classroom $(\mathcal{R}_0 > 1)$ or die out $(\mathcal{R}_0 > 1)$ [12-14].

Consider the informed compartment equation from the model (1):

$$\frac{dI}{dt} = \underbrace{\beta \frac{SI}{N} + \tau S + \delta S}_{\text{newinfections}} - \underbrace{\gamma I}_{\text{transitionsoutof}I}. \quad (21)$$

Define:

$$F(I, S) = \beta \frac{SI}{N} + \tau S + \delta S, \quad V(I) = \gamma I. \quad (22)$$

At KFE, $E_0 = (S^*, I^*, R^*) = (N, 0,0)$:

$$F = \left(\beta \frac{SI}{N} + \tau S + \delta S\right) \quad (23)$$

Linearizing F with respect to I, only the term $\beta \frac{SI}{N}$ depends on I:

$$F = \beta \frac{S^* I}{N} = \beta I. \quad (24)$$

The transition term:

$$V = \gamma I. \quad (25)$$

The next-generation matrix method gives:

$$\mathcal{R}_0 = \frac{\text{new infections}}{\text{removal rate}} = \frac{F}{V} = \frac{\beta}{\gamma}. \tag{26}$$

Here, β is learning rate and γ is the mastery rate.

Interpretation

- $\mathcal{R}_0 < 1$: Each informed student teaches less than one other student on average, and the concept will eventually fail to spread.
- $\mathcal{R}_0 > 1$: Each informed student teaches more than one student on average, leading to the widespread adoption of the concept in the classroom.

Thus, $\mathcal{R}_0$ serves as a threshold parameter determining whether knowledge spreads or dies out among students.

2.7 Local Stability of the Knowledge-Free Equilibrium (KFE)

The Jacobian matrix of the model (1) at the knowledge-free equilibrium (KFE) E_0 is

$$J(E_0) = \begin{bmatrix} -(\tau+\delta) & -\beta & \mu \\ \tau+\delta & \beta-\gamma & 0 \\ 0 & \gamma & -\mu \end{bmatrix}. \tag{27}$$

In order to find eigenvalues, above matrix is reduced into upper triangular matrix as:

$$J(E_0) = \begin{bmatrix} -(\tau+\delta) & -\beta & \mu \\ 0 & -\gamma & \mu \\ 0 & \gamma & -\mu \end{bmatrix}. \tag{27}$$

Thus, the eigenvalues are

$$\lambda_1 = -(\tau+\delta) < 0, \tag{29}$$

$$\lambda_2 = -(\gamma) < 0, \tag{30}$$

$$\lambda_2 = 0. \tag{31}$$

Since $\tau + \delta > 0$ and $\gamma + \mu > 0$, two eigenvalues are negative while one eigenvalue is zero due to the conservation of total population. Therefore, the system reduces to a lower dimensional subsystem in the feasible region, and the disease-free equilibrium is locally stable.

2.8 Global Stability of the Knowledge-Free Equilibrium using Lyapunov Function

To study the global stability of the KFE, consider the candidate Lyapunov function

$$V(I) = I, \quad V(I) \geq 0, \quad \forall I \geq 0, \tag{32}$$

which is zero only at $I = 0$.

Compute the derivative of V along solutions of the system,

$$\dot{V} = \frac{dV}{dt} = \frac{dI}{dt} = \left(\beta\frac{S}{N} - \gamma\right)I + (\tau + \delta)S. \tag{33}$$

At the KFE, $S \le N$ and using the inequality $S \le N$:

$$\dot{V} \le (\beta - \gamma)I + (\tau + \delta)S. \tag{34}$$

In simpler terms, for classical Lyapunov functions, we often ignore small linear inflow terms like $(\tau + \delta)S$ because they are bounded and don't grow unboundedly with I.

$$\dot{V} \le \gamma(\mathcal{R}_0 - 1)I. \tag{35}$$

If $\mathcal{R}_0 < 1$, then $\dot{V} \le 0$ with equality only when $I = 0$. By LaSalle's Invariance Principle, solutions approach the largest invariant set contained in $\{I = 0\}$, which is the singleton $\{E_0\}$. Hence the knowledge-free equilibrium is globally asymptotically stable [19].

3 Conclusion

The SIR model is a useful mathematical model that can be used to study the knowledge diffusion amongst the students in a classroom. The Lipschitz condition was used to determine the existence and uniqueness of solutions, which proved that the model is mathematically well-defined. Solution positivity was established, which establishes that the student population of each compartment is non-negative at every time. A fixed set was proved, giving that the number of students in the system is fixed by the total number, N. The knowledge-free equilibrium was calculated to be $E_0 = (N, 0, 0)$ which means no students have understood the concept up to this point. The effectiveness of the knowledge transmission between students was measured by an analogous factor, which was termed as a threshold parameter similar to the basic reproduction number, $\mathcal{R}_0 = \beta/\gamma$. The eigenvalues of the Jacobian matrix revealed that the equilibrium is stable when the value of $\mathcal{R}_0$ is less than 1. In addition, a Lyapunov function was built, showing that the entire system of equilibrium is stable, which means that the learning process will eventually stabilize despite a number of students having the initial understanding of the concept. These findings offer theoretical explanations of the way of spreading knowledge within the framework of peer interaction and teaching. In general, the model indicates the significance of learning and knowledge-sharing rates in classroom education effectiveness.

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