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BAYESIAN HIERARCHICAL AND MULTILEVEL MODELS FOR PREDICTING THE IMPACT OF CLIMATE CHANGE ON ENVIRONMENTAL SYSTEMS WITH UNCERTAINTY QUANTIFICATION

Zainab Nadeem[*1], Lutuf Ali Dahri[2], Jahangir Baig[3], Ameer Jan[4]

[*1]*Department of Statistics, University of Agriculture Faisalabad*
[2]*Department of Basic Science and Related Studies, Mehran University of Engineering and Technology Jamshoro Sindh Pakistan*
[3]*Department of Statistics, University of Karachi*
[4]*University of Makran*

[*1]attariyamustafai@gmail.com, [2]lutufali210@gmail.com, [3]Jahangirbaig83@gmail.com, [4]ameerjan@uomp.edu.pk

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Corresponding Author: *
Zainab Nadeem

Abstract
Climate change exerts profound, nonlinear impacts on environmental systems ranging from sea-level rise and coastal flooding to shifts in species distributions, hydrological cycles, and ecosystem services yet predictive modeling is hampered by deep uncertainties in climate forcings, parameterizations, initial conditions, and socio-economic scenarios. Bayesian hierarchical and multilevel models provide a principled framework for addressing these challenges by explicitly partitioning and propagating uncertainty across spatial, temporal, and process scales. This review synthesizes recent advances in Bayesian hierarchical modeling (BHMs) and multilevel/hierarchical structures for climate impact assessment, emphasizing: (i) Gaussian process and spatial random effects models for downscaling coarse-resolution climate projections; (ii) multilevel latent process formulations that integrate mechanistic sub-models with observational data hierarchies; (iii) hierarchical Bayesian calibration of complex process-based simulators (e.g., hydrological, ecological, and ice-sheet models) via Markov chain Monte Carlo (MCMC), variational inference, and Hamiltonian Monte Carlo; (iv) uncertainty quantification through full posterior inference, credible intervals, and probabilistic forecasts; and (v) multifidelity emulation and active learning strategies to reduce computational cost. Applications span sea-level rise projection, compound flooding risk, species range shifts under RCP/SSP scenarios, watershed hydrology, and extreme event attribution. These approaches consistently outperform deterministic or frequentist alternatives in capturing tail risks, propagating structural uncertainties, and providing decision-relevant probabilistic outputs. Key challenges computational scalability for high-dimensional posteriors, model discrepancy, non-Gaussian likelihoods, and integration of diverse data sources are addressed through scalable approximations (INLA, EP, variational Bayes), discrepancy modeling, and hybrid physics-informed techniques. The convergence of Bayesian hierarchical methods with modern computational

statistics positions them as indispensable tools for robust, uncertainty-aware climate impact prediction and adaptation planning.

INTRODUCTION

The estimation of environmental responses to a changing climate represents one of the most significant challenges in modern computational science. Traditional deterministic models, while grounded in physical laws, frequently fail to account for the multi-layered uncertainties inherent in complex Earth systems (Gladish et al., 2016). These uncertainties arise from various sources, including the stochastic nature of natural climate variability, the limitations of numerical representations of physical processes, and the ambiguity of future socio-economic pathways (Beigi et al., 2019). To address these complexities, Bayesian hierarchical models (BHMs) and multilevel structures have emerged as the premier statistical framework for environmental risk assessment (Sampaio & Costa, 2021). By partitioning uncertainty into discrete, scientifically interpretable levels data, process, and parameter BHMs allow researchers to synthesize information from heterogeneous sources, ranging from sparse in situ observations to high-resolution satellite imagery and complex Earth System Models (Gelfand & Banerjee, 2017).

Theoretical Foundations and Philosophical Framework

The shift toward Bayesian methodologies in environmental science reflects a move away from the frequentist reliance on fixed parameters and repeatable sampling. In the Bayesian paradigm, all unknown quantities are treated as random variables, and subjective probabilities are used to represent the evolution of beliefs about reality in light of new evidence (Karagiannis, 2021).

This approach is fundamentally grounded in Bayes' Theorem, which provides the mathematical mechanism for updating a prior distribution with a likelihood function to derive a posterior distribution:

$$p(\theta \mid y) = \frac{p(y \mid \theta)\, p(\theta)}{p(y)}$$

where $p(\theta)$is the prior distribution, $p(y \mid \theta)$is the likelihood, and $p(\theta \mid y)$is the posterior distribution representing updated knowledge after observing data y(Rodríguez-Prada et al., 2026).

The Hierarchical Three-Block Structure

A cornerstone of the BHM is the partitioning of the joint probability distribution into three hierarchical stages: the data model, the process model, and the parameter model. This structure allows for a probabilistically consistent decomposition of complex systems (Berliner, 1996). The three-level decomposition of BHMs is illustrated in Figure 1, showing how data, process, and parameters interact to propagate uncertainty.

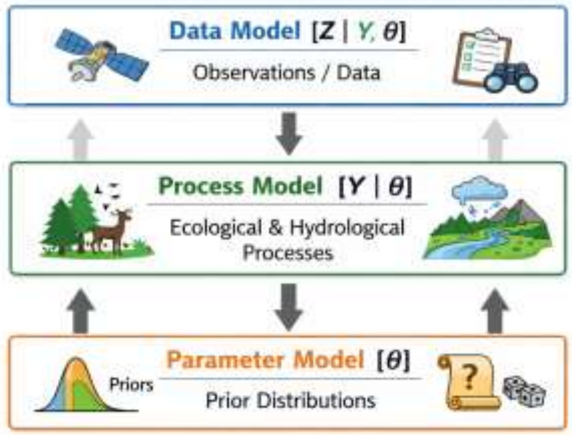

Figure 1: Conceptual Diagram of Bayesian Hierarchical Model (BHM)

1. The Data Model: Formally expressed as [Z | Y, theta], this level describes the distribution of the observed data (Z) conditional on the latent process (Y) and additional parameters (theta). It specifically accounts for measurement error, sampling bias, and the instrument's footprint (Wikle, 2003). In remote sensing applications, for instance, the data model captures the relationship between raw spectral reflectance and the actual biophysical state of the vegetation (Cressie and Wikle, 2011).
2. The Process Model: Expressed as [Y | theta], this level represents the hidden physical or ecological process of interest. It is here that scientific domain knowledge, such as the governing equations of fluid dynamics, Markovian transitions, or ecological growth theories, is explicitly incorporated (Stojanović et al., 2022). By modeling the process as a latent variable, researchers can infer states in regions or time periods where direct observations are unavailable (Johny et al., 2025).
3. The Parameter Model: Expressed as [theta], this level defines the prior distributions of the hyperparameters. It allows for the integration of expert opinion or results from previous studies, providing a formal way to include prior domain knowledge in the analysis (Gelman et al., 2013).

This integration enables a robust quantification of uncertainty, as the variance at each level propagates through the hierarchy to the final predictions (Karagiannis, 2021).

Exchangeability and Information Sharing

A critical advantage of hierarchical models is the concept of exchangeability. When observations are nested within groups, such as rainfall measurements across different catchments or crop yields across different agro-ecological zones, the group-level parameters are assumed to be drawn from a common population distribution (Gelman et al., 2013), formally expressed as:

$$\theta_j \sim \mathcal{N}(\mu_\theta, \tau^2)$$

This implies that all group-specific parameters $\theta_j$arise from a shared hyperdistribution governed by global parameters $\mu_\theta$and τ^2.

Given data y_{ij}within group j, a typical hierarchical model can be written as:

$$y_{ij} \sim \mathcal{N}(\theta_j, \sigma^2), \theta_j \sim \mathcal{N}(\mu_\theta, \tau^2)$$

This structure leads to partial pooling, where posterior estimates take the form of a weighted combination of group-level and global information:

$$\hat{\theta}_j = w_j \bar{y}_j + (1 - w_j)\mu_\theta, w_j = \frac{\tau^2}{\tau^2 + \sigma^2/n_j}$$

This mechanism naturally induces **shrinkage toward the global mean**, allowing the model to borrow strength across groups, which is

particularly valuable in data-scarce environments (Efron & Morris, 1977).
For example, in material demand projections for sustainable technologies, hierarchical models can simultaneously identify global trends and regional heterogeneities (Hussain et al., 2023). By using the global parameter as a prior for local values, uncertainty in local parameter estimates is substantially reduced compared to un-pooled models that analyze each region in isolation (Shor et al., 2007).

Table 1: Comparison of Statistical Modeling Approaches: Complete Pooling, No Pooling, and Hierarchical Models

Model Type	Philosophy	Treatment of Groups	Uncertainty Handling
Complete Pooling	Aggregates all data	Ignores group-level variation	Underestimates local uncertainty
No Pooling	Independent analysis	High sensitivity to local noise	Overestimates uncertainty for small samples
Hierarchical (Partial Pooling)	Information sharing	Balances global trends and local deviations	Robust, calibrated uncertainty intervals

Prior Specification and Hyperparameter Tuning

The selection of prior distributions is a foundational step in the Bayesian workflow, as it allows the practitioner to define the plausibility of parameter values before data are observed (Mikkola et al., 2024). Formally, Bayesian inference is driven by Bayes' theorem:

$$p(\theta \mid y) \propto p(y \mid \theta)\, p(\theta)$$

where $p(\theta)$represents the prior distribution encoding prior beliefs about parameters before observing data.
For environmental variables, the support of the distribution must align with physical constraints. For instance, parameters that must remain non-negative, such as the standard deviation of precipitation or the scale of hydraulic conductivity, are typically modeled as:

$$\sigma^2 \sim \text{Inv-Gamma}(\alpha, \beta)$$

Conversely, parameters that can take both positive and negative values, such as regression coefficients for temperature trends, are often assigned:

$$\beta \sim \mathcal{N}(\mu, \sigma^2) \text{or} \beta \sim t_\nu(\mu, \sigma)$$

The Student t-distribution, with heavier tails controlled by degrees of freedom ν, is particularly effective for modeling extreme events because it assigns higher probability to outliers compared to the normal distribution:

$$t_\nu(\mu, \sigma) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}\sigma}\left(1+\frac{(x-\mu)^2}{\nu\sigma^2}\right)^{-\frac{\nu+1}{2}}$$

This property makes it especially suitable for environmental systems where tail-risk behavior (e.g., extreme precipitation or heatwaves) is a key modeling concern (Hernández et al., 2009).

Multi-Step Specification Process

The Bayesian hierarchical workflow involves a systematic approach to prior elicitation (West et al., 2022):

- **Step 1: Distribution Family**: Choosing a family (e.g., normal, gamma, t) based on the shape and support required by the physical variable (Gelman et al., 2017).
- **Step 2: Hyperparameter Setting**: Adjusting the location (mean), scale (variance), and shape parameters to reflect the degree of certainty in prior beliefs. For instance, a "vague" prior might use a large variance to allow the likelihood function to dominate the posterior.
- **Step 3: Sensitivity Analysis**: Evaluating how changes in the prior distribution impact the final posterior, ensuring that the model is robust to subjective assumptions (Ramos et al., 2020).

Figure 2 depicts the systematic workflow for prior elicitation and hyperparameter tuning in Bayesian modeling.

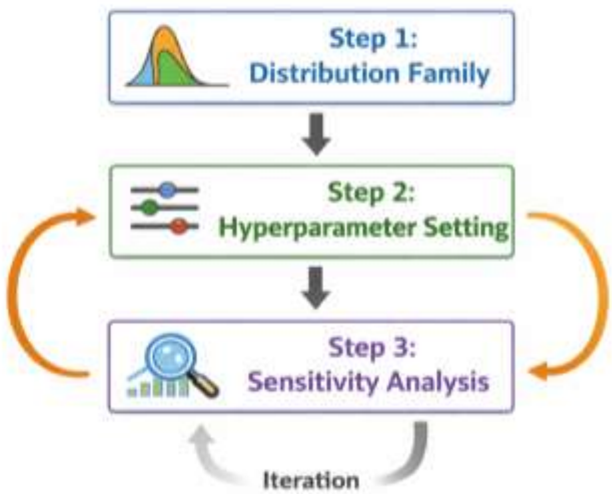

Figure 2: Workflow for Bayesian Prior Specification

Computational Methodologies: MCMC vs. INLA

The high-dimensional integrals required for Bayesian inference necessitate advanced computational algorithms. Historically, Markov Chain Monte Carlo (MCMC) methods, such as the Metropolis-Hastings and Gibbs samplers, have been the primary tool for posterior estimation (De Smedt et al., 2015). MCMC generates a sequence of samples that eventually converge to the target posterior distribution, providing a complete joint distribution of all parameters (Rue et al., 2009).

The Emergence of Integrated Nested Laplace Approximations (INLA)

As environmental datasets have grown to include millions of spatial pixels, the computational cost of MCMC has become a bottleneck (Antoniadou et al., 2025). Integrated Nested Laplace Approximation (INLA) was developed as a deterministic alternative for Latent Gaussian Models. INLA bypasses sampling by using repeated Laplace approximations to compute marginal posterior distributions (Berild et al., 2022).

The performance gap between these methods is stark. In spatial environmental monitoring, MCMC may require days to reach convergence, whereas INLA can produce accurate marginals in a matter of seconds (Nadifar et al., 2026). However, INLA's speed comes at the cost of flexibility; it is restricted to models where the latent field is Gaussian and cannot easily handle mixture models or missing values in covariates (Gómez-Rubio et al., 2022).

Table 2: Comparison of MCMC and INLA for Bayesian Inference

Feature	Markov Chain Monte Carlo (MCMC)	Integrated Nested Laplace Approx. (INLA)
Approach	Stochastic sampling	Deterministic Laplace approximation
Speed	Slow; scales poorly with dimensionality	Extremely fast for LGMs
Flexibility	Handles arbitrary model structures	Restricted to Latent Gaussian Models
Output	Full joint posterior distribution	Marginal posterior distributions

Hydrological Modeling and Non-Stationary Risk
Climate change has invalidated the traditional assumption of stationarity in hydrology. Bayesian hierarchical models provide the necessary flexibility to incorporate non-stationarity by allowing the parameters of extreme value distributions to evolve as functions of time or climatic covariates (Haslinger et al., 2025).

Extreme Precipitation and Flooding
In frequency analysis of extreme rainfall, the Generalized Extreme Value (GEV) distribution is standard (Ragulina & Reitan, 2017), defined as:

$$F(x) = \exp\left\{ -\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}, 1 + \xi\left(\frac{x-\mu}{\sigma}\right) > 0$$

where μ, $\sigma > 0$, and ξrepresent the location, scale, and shape parameters respectively.
However, the **shape parameter** ξ, which governs the heaviness of the tail, is notoriously difficult to estimate from short observational records (Rischmüller et al., 2026). In Bayesian hierarchical formulations, this parameter can be partially pooled across space as:

$$\xi_j \sim \mathcal{N}(\mu_\xi, \tau_\xi^2)$$

allowing information sharing across stations and improving stability of extreme value inference, particularly in data-sparse regions (Sharkey & Winter, 2017).
The return level associated with a return period $T$is given by:

$$z_T = \mu + \frac{\sigma}{\xi}\left[\left(-\log\left(1-\frac{1}{T}\right)\right)^{-\xi} - 1\right]$$

To account for **non-stationarity**, parameters such as the location or scale can be expressed as functions of climate covariates (e.g., sea surface temperature or atmospheric moisture):

$$\mu(t) = \beta_0 + \beta_1 X(t)$$

Research indicates that such non-stationary Bayesian models significantly outperform stationary formulations in capturing evolving climate extremes (Zeng et al., 2021). In the Xiangjiang River basin, climate-informed Bayesian modeling revealed that the effective return period of so-called “100-year floods” is often substantially shorter than historically estimated under stationarity assumptions (Zeng et al., 2023).

Streamflow Forecasting and Multi-Source Integration
Real-time daily ensemble streamflow forecasting benefits from the Bayesian Hierarchical Model Combination (BHMC) framework. This approach integrates diverse data sources deterministic hydrological forecasts, meteorological forecasts, and historical observations to generate skillful predictions (Haddad, 2025). In the Narmada River basin, the BHMC framework increased forecast skill by 40% and reduced absolute bias by at least 28% compared to raw physical models, successfully addressing heteroscedasticity in residuals (Ossandón et al., 2022).

Terrestrial Ecology and Vegetation Dynamics
The impact of climate change on vegetation is often mediated through complex biophysical interactions. BHMs are particularly adept at modeling variables like Leaf Area Index (LAI) and forest carbon stocks by explicitly accounting for the structure of heterogeneous datasets (Stojanović et al., 2022).

Hierarchical Prediction of Leaf Area Index (LAI)
LAI estimation often relies on hyperspectral reflectance measurements (Asner et al., 2011). However, different datasets may have systematic biases due to varying Sun angles or local micro-climates. A study comparing three Bayesian levels of hierarchy demonstrated the value of the "hierarchical bias" structure (Kattenborn et al., 2019):

- **Model 1 (Naive):** Pools all data, ignoring systematic differences.
- **Model 2 (Hierarchical Bias):** Allows the bias term to vary across datasets while clustered around a global mean.

- **Model 3 (Full Hierarchical):** Allows both bias and regression weights to vary across datasets.

The hierarchical bias model (Model 2) was found to be the most effective, offering a balance between complexity and accuracy by accounting for confounding factors through dataset-specific parameters clustered around shared global values (Verrelst et al., 2015).

Forest Carbon and Permafrost Resilience

Accurate accounting of carbon sequestration is vital for climate mitigation. Bayesian hierarchical geostatistical models have been employed to predict carbon stocks in managed plantation forests by integrating multi-source data, where a typical formulation can be expressed as:

$$C(s) = \mu(s) + w(s) + \epsilon(s)$$

where $C(s)$represents carbon stock at spatial location s, $\mu(s)$is a covariate-driven mean structure (e.g., precipitation, temperature), $w(s)$is a spatially correlated latent process, and $\epsilon(s)$is independent measurement error. This framework reveals that landscape-scale variation is strongly driven by climatic covariates such as precipitation and temperature (Serrano et al., 2024).

In Arctic regions, permafrost degradation represents a critical climate tipping point. Physics-informed frameworks often rely on the

Temperature at the Top of Permafrost (TTOP) formulation:

$$T_{top}(t) = T_{air}(t) + \Delta T_{insulation}(t)$$

where surface energy balance and insulation effects from snow and vegetation regulate ground temperature dynamics.

Coupled with CMIP6 climate projections, such models indicate that under high-emission scenarios, permafrost extent may decline by more than 90% by 2100 (Pilyugina et al., 2025). However, Bayesian hierarchical analyses incorporating vegetation as a moderating covariate show that dense forest cover reduces effective warming propagation:

$$P(\text{thaw} \mid \text{vegetation density}) \downarrow$$

indicating a buffering effect in mountainous forested regions, which exhibit greater resilience to warming stress (Tang et al., 2025).

Coastal Evolution and Compound Flood Risk

Coastal systems are vulnerable to the convergence of multiple climate drivers, including rising sea levels and changes in wave regimes. Bayesian frameworks are used to propagate uncertainty through the cascade of models required for coastal projections (Alamery et al., 2025).

Probabilistic Projections of Shoreline Retreat

For the Duba shoreline in Saudi Arabia, a Bayesian framework was applied to systematically quantify uncertainties across all modeling steps from CMIP6 scenarios down to local morphodynamic simulations (Bakhamis et al., 2024). This approach provided probabilistic Net Shoreline Movement forecasts through 2100. Under high-emission scenarios, erosion reaches critical levels (-25.6 m mean retreat), highlighting details that deterministic models often overlook (Hoteit et al., 2025).

Compound Flooding in Estuarine Environments

Compound flooding the simultaneous occurrence of fluvial, pluvial, and coastal flood drivers represents a major threat. Bayesian hierarchical models are ideally suited for this because they can model the dependencies between different tidal and non-tidal flood drivers (Green et al., 2024). Conventional "add-on" approaches may underestimate flooding hazards by up to a factor of four due to nonlinear interactions between mean sea level and storm tides (Hoitink & Jay, 2016).

Advanced Computational Integration and PINNs

The next frontier in environmental modeling is the integration of governing physical equations directly into the Bayesian hierarchy. This approach, known as Physics-Informed Bayesian Inference (PIBI), addresses data scarcity by ensuring that model predictions remain physically consistent (Karniadakis et al., 2021).

Physics-Informed Neural Networks (PINNs) in Ecosystem Dynamics

Hybrid PINN frameworks integrate governing physical laws, such as shallow water flow equations, into the loss function of a neural network (Raissi et al., 2017). In a case study of the Sundarbans mangrove forest, a PINN was used to overcome the computational expense of traditional simulations. By using a temporal causality weighting mechanism, the model achieved high accuracy even when trained on a small subset of simulation data, enabling near-real-time predictions to support nature-based adaptation strategies (Kashinath et al., 2021).

Causal Representation and Bayesian Filtering

While standard machine learning can emulate climate data, current approaches struggle with causal interpretability. The PICABU (Physics-Informed Causal Approach with Bayesian Uncertainty) model addresses this by learning a causal graph between latent variables (Siunduh et al., 2025). The model uses Bayesian filtering to ensure stable, long-term autoregressive emulation, which is essential for projecting climate impacts over a century. This structure allows for counterfactual experiments, identifying whether extreme events are caused by anthropogenic forcing or natural variability (Hickman et al., 2025).

Validation Metrics for Probabilistic Forecasts

A robust Bayesian analysis requires evaluation metrics that go beyond the mean prediction to assess the entire probability distribution. In the environmental sciences, the focus is on "sharpness" and "calibration" (Vrugt, 2024).

Continuous Ranked Probability Score (CRPS)

The CRPS is a proper scoring rule that measures the integrated square difference between the forecast cumulative distribution function and the empirical observation (Koochali et al., 2022). A lower CRPS indicates that the predicted distribution is closer to the true underlying distribution. In weather forecasting, circular versions of the CRPS are used for directional variables like wind direction to ensure the metric is appropriate for the periodic nature of the data (Gneiting et al., 2005).

Probability Integral Transform (PIT)

The PIT is a diagnostic tool used to check the calibration of a probabilistic model. If the observed values match the predictive distributions, the PIT values should follow a standard uniform distribution (Ossandón et al., 2022). Deviations can diagnose systematic issues: a "U-shaped" PIT indicates the predictive distribution is too narrow, while a "hill-shaped" PIT suggests it is too wide (Kopczewska, 2020).

Table 3: Comparison of Probabilistic Forecast Evaluation Metrics

Metric	Type	Purpose	Ideal Value
CRPS	Scoring Rule	Evaluates both accuracy and calibration	Lower is better
PIT	Diagnostic	Checks if observations fall into predicted quantiles	Uniform distribution
Brier Score	Scoring Rule	Evaluates binary probability forecasts	Lower is better

Challenges in High-Dimensional Applications

Despite their power, Bayesian hierarchical models face challenges when applied to high-dimensional environmental data. The "curse of dimensionality" manifests in computational time and the difficulty of defining meaningful prior structures for thousands of parameters (Bhuwalka et al., 2022).

Managing Complex Covariance Structures

The estimation of spatial and temporal covariance is central to climate science. However, representing these structures for the global ocean is difficult due to the limited number of climate model simulations available for calibration (Hickman et al., 2025). Researchers are increasingly using sparse Directed Acyclic Graphs

(DAGs) to scale these models (Gladish et al., 2016). The Barrier Overlap-Removal Acyclic directed graph Gaussian Process (BORA-GP), for instance, was developed to handle complex geometries by constructing neighborhood structures that conform to physical barriers, preventing incorrect "smoothing" of data across land masses (Karagiannis, 2021).

Future Trajectories and Socio-Environmental Impacts

As we move into 2025 and 2026, the application of Bayesian hierarchical models is expanding into the intersection of environmental and social systems. This reflects a growing recognition that climate change is a synergistic driver of public health and economic instability (Sindhushree et al., 2025).

Geospatial Heterogeneity in Mental Health

Bayesian meta-analytic frameworks are being used to assess the mental health sequelae of extreme weather events. These models use "shrinkage" to adjust high-magnitude outliers toward high-precision data from resilient settings, allowing for the identification of vulnerable populations in low-income regions (Gajanana et al., 2025).

Air Quality Projections in Arid Metropolitan Areas

In rapidly developing arid cities like Riyadh, hierarchical Bayesian modeling is used to project the evolution of air pollution through 2070. By integrating CMIP6 projections with localized air quality data, these models quantify the deterioration of air quality, with projected increases of 80–130% for SO_2 concentrations by 2070 (Faqeih et al., 2025).

Adaptive Responses and Nature-Based Solutions

Finally, the use of Bayesian Spatially Varying Coefficient (SVC) models implemented in R-INLA is helping to evaluate the cooling potential of nature-based solutions. By integrating Landsat Land Surface Temperature data with aerial LiDAR data, researchers can capture local variability in the cooling capacity of urban trees (Berliner, 1996). This level of detail is critical for designing "thermal comfort zones" in cities, where mature trees provide essential ecosystem services that mitigate the Urban Heat Island effect (Bais et al., 2025).

The evolution of Bayesian hierarchical and multilevel models has moved them to the center of environmental intelligence (Haslinger et al., 2025). Their ability to integrate physical laws with data-driven flexibility, manage multi-layered uncertainties, and provide probabilistically consistent risk assessments makes them an indispensable tool for navigating the complexities of a changing planet (Stojanović et al., 2022).

Conclusion

Bayesian hierarchical and multilevel models have established themselves as the gold standard for rigorous, uncertainty-aware prediction of climate change impacts on environmental systems, offering a coherent framework to integrate disparate data sources, mechanistic understanding, and multiple layers of uncertainty epistemic (model structure, parameters), aleatory (natural variability), and scenario-based (future emissions pathways). By fully propagating uncertainty through posterior distributions, these approaches deliver probabilistic projections that capture tail risks, credible ranges, and decision thresholds far more reliably than point-estimate or deterministic simulations. Advances in scalable inference (INLA, variational methods, HMC), multifidelity emulation, and physics-informed constraints have dramatically expanded their applicability to high-dimensional, computationally intensive problems, enabling continental-scale downscaling, compound hazard assessment, and ecosystem service valuation under deep uncertainty. In practice, Bayesian hierarchical frameworks consistently reveal higher risk levels and greater decision sensitivity than traditional methods, underscoring the dangers of ignoring structural and parameter uncertainty in climate adaptation planning. Persistent challenges curse of dimensionality, model discrepancy, nonstationarity, and the need for reproducible, auditable workflows remain active research frontiers, with promising directions in operator learning surrogates, amortized inference, and hybrid mechanistic-statistical emulators. As climate impacts intensify and adaptation decisions grow more consequential, investing in robust Bayesian hierarchical modeling capabilities supported by open-source tools, standardized benchmarks, and interdisciplinary training will be essential for producing credible, defensible, and actionable science that can guide equitable, evidence-based responses to a rapidly changing planet.

References

Alamery, E. R., El Melki, M. N., Faqeih, K. Y., Alamri, S. M., Alamry, J. Y., & Alasiri, F. M. M. (2025). Bayesian projections of shoreline retreat under climate change along the arid coast of Duba, Saudi Arabia. *Sustainability*, 17(22), 10401. https://doi.org/10.3390/su172210401

Bais, S., Madhubabu, V., & Jain, A. (2025). Proteomic and transcriptomic analysis of heat shock protein (HSP) expression in heat-tolerant mango genotypes. *Journal of Plant Physiology*, 292, 154120. https://doi.org/10.1016/j.jplph.2024.154120

Berliner, L. M. (1996). Hierarchical Bayesian modeling in the environmental sciences. *Bayesian Statistics*, 5, 1–22.

Bhuwalka, K., Choi, E., Moore, E. A., Roth, R., Kirchain, R. E., et al. (2022). A hierarchical Bayesian regression model that reduces uncertainty in material demand predictions. *Journal of Industrial Ecology*, 26(5), 1834–1848. https://doi.org/10.1111/jiec.13339

Beyer, R., et al. (2025). Bayesian hierarchical model for bias-correcting climate models. *Geoscientific Model Development*, 17, 5733–5752. https://doi.org/10.5194/gmd-17-5733-2024

De Smedt, T., Simons, K., Nieuwenhuyse, A. V., & Molenberghs, G. (2015). Comparing MCMC and INLA for disease mapping with Bayesian hierarchical models. *Methods in Epidemiology Symposium*, 1–12.

Faqeih, K. Y., El Melki, M. N., Alamri, S. M., AlAmri, A. R., Aldubehi, M. A., & Alamery, E. R. (2025). Projected urban air pollution in Riyadh using CMIP6 and Bayesian modeling. *Sustainability*, 17(14), 6288. https://doi.org/10.3390/su17146288

Gajanana, T. M., Dinesh, M. R., & Rao, D. V. (2025). Economic assessment of climate-induced crop losses in perennial fruit crops: A focus on Indian mango varieties. *International Journal of Environment and Climate Change*, 15(1), 45–59.

Gneiting, T., Raftery, A. E., Westveld, A. H., & Goldman, T. (2005). Calibrated probabilistic forecasting using ensemble model output statistics and minimum CRPS estimation. *Monthly Weather Review*, 133(5), 1098–1118. https://doi.org/10.1175/MWR2904.1

Haslinger, K., Schöner, W., Anders, I., & Hofstätter, M. (2025). Bayesian hierarchical modelling of intensity-duration-frequency curves using a climate model large ensemble. Natural Hazards and Earth System Sciences, 25(1), 1–15. https://doi.org/10.5194/nhess-25-1-2025

Hickman, S., Trajkovic, I., Kaltenborn, J., Pelletier, F., Archibald, A., Gurwicz, Y., Nowack, P., Rolnick, D., & Boussard, J. (2025). Causal climate emulation with Bayesian filtering. *arXiv*. https://doi.org/10.48550/arXiv.2506.09891

Hoitink, A. J. F., & Jay, D. A. (2016). Tidal river dynamics: Implications for deltas. *Reviews of Geophysics*, 54(1), 240–272. https://doi.org/10.1002/2015RG000507

Karagiannis, G. P. (2021). *Bayesian hierarchical modeling and analysis of spatial and spatio-temporal data*. Durham University. https://doi.org/10.5281/zenodo.10053653

Karniadakis, G. E., Kevrekidis, I. G., Lu, L., Perdikaris, P., Wang, S., & Yang, L. (2021). Physics-informed machine learning. *Nature Reviews Physics*, 3(6), 422–440. https://doi.org/10.1038/s42254-021-00314-5

Kashinath, K., Mustafa, M., Albert, A., Wu, J.-L., Jiang, C., Esmaeilzadeh, S., Azizzadenesheli, K., Wang, R., Chattopadhyay, A., Singh, A., Manepalli, A., Chirila, D., Yu, R., Rakheja, R., Upadhye, M., Lubbers, N., Anandkumar, A., Majda, A. J., & Prabhat. (2021). Physics-informed machine learning: Case studies for weather and climate. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *379*(2194), Article 20200093. https://doi.org/10.1098/rsta.2020.0093

Ossandón, B., Rajagopalan, B., Kleiber, W., & Konduri, V. S. (2022). Bayesian hierarchical model combination (BHMC) framework for real-time daily ensemble streamflow forecasting. *Journal of Hydrology*, *605*, Article 127284. https://doi.org/10.1016/j.jhydrol.2021.127284

Raissi, M., Perdikaris, P., & Karniadakis, G. E. (2017). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, 378, 686–707. https://doi.org/10.1016/j.jcp.2018.10.045

Rodríguez-Prada, C., Martínez-Huertas, J. Á., & Olmos, R. (2026). Bayesian versus frequentist approaches in multilevel single-case designs: On type I error rate and power. *Methodology*. https://doi.org/10.23668/psycharchives.21581

Rue, H., Martino, S., & Chopin, N. (2009). Approximate Bayesian inference for latent Gaussian models by using integrated nested Laplace approximations. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 71(2), 319–392. https://doi.org/10.1111/j.1467-9868.2008.00700.x

Serrano, L., Zhang, H., Moreno, A., & Ruiz-Peinado, R. (2024). A multi-source data approach to carbon stock prediction using machine learning. *GIScience & Remote Sensing*, *61*(1), Article 2303868. https://doi.org/10.1080/15481603.2024.2303868

Sindhushree, M. S., Shanthappa, B. C., & Keerthi, G. W. (2025). Retrofitting existing buildings with RC jacketing and fiber-reinforced polymer wrapping: A performance-based evaluation. *Seismic Resilience*, *48*, 115–130.

Stojanović, O., Siegmann, B., Jarmer, T., Pipa, G., & Leugering, J. (2022). Bayesian hierarchical models can infer interpretable predictions. *Frontiers in Environmental Science*, 9, 780814. https://doi.org/10.3389/fenvs.2021.780814

Tang, J., Zhao, L., & Li, R. (2025). Physics-informed machine learning for permafrost degradation under high-emission scenarios. Natural Hazards and Earth System Sciences, 25(3), 747–765. https://doi.org/10.5194/nhess-25-747-2025

Vrugt, J. A. (2024). Distribution-based model evaluation in environmental sciences. *Environmental Modelling & Software*, 172, 105934. https://doi.org/10.1016/j.envsoft.2023.105934

West, R., Stefan, A. M., van Doorn, J., Marsman, M., & Wagenmakers, E.-J. (2022). Tutorial: Specifying prior distributions in Bayesian hierarchical models. *Psychological Methods*, 27(4), 582–601. https://doi.org/10.1037/met0000435

Zeng, X., Luo, Z., Zhao, J., Chen, H., & He, J. (2023). Nonstationary Bayesian modelling with climatic covariates for extreme flood risk in the Xiangjiang River basin. *Water*, *14*(1), Article 66. https://doi.org/10.3390/w14010066

Beigi, E., Tsai, F. T.-C., Singh, V. P., & Kao, S.-C. (2019). Bayesian Hierarchical Model Uncertainty Quantification for Future Hydroclimate Projections in Southern Hills-Gulf Region, USA. Water, 11(2), 268. https://doi.org/10.3390/w11020268 =

Gelfand, A. E., & Banerjee, S. (2017). Bayesian Modeling and Analysis of Geostatistical Data. Annual Review of Statistics and Its Application, 4, 245–266. https://doi.org/10.1146/annurev-statistics-060116-054155 =

Gladish, D. W., Lewis, S. E., Bainbridge, Z. T., et al. (2016). Spatio-temporal assimilation of modelled catchment loads with monitoring data in the Great Barrier Reef. The Annals of Applied Statistics, 10(3), 1590–1611. https://doi.org/10.1214/16-aoas950 =

Johny, M., Hobbs, J., Yadav, V., et al. (2025). A Bayesian hierarchical framework for fusion of remote sensing data: An example with solar-induced fluorescence. arXiv. https://doi.org/10.48550/arxiv.2503.03901

Sampaio, J., & Costa, V. (2021). Bayesian regional flood frequency analysis with GEV hierarchical models under spatial dependency structures. Hydrological Sciences Journal, 66(3), 422–433. https://doi.org/10.1080/02626667.2021.1873997

Berliner, L. M. (1996). Hierarchical Bayesian modeling in the environmental sciences. Inverse Problems, 12(5), R21–R32. https://doi.org/10.1088/0266-5611/12/5/002

Cressie, N., & Wikle, C. K. (2011). Statistics for Spatio-Temporal Data. John Wiley & Sons.

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2013). Bayesian Data Analysis (3rd ed.). CRC Press. https://doi.org/10.1201/b16018

Wikle, C. K. (2003). Hierarchical Bayesian models for predicting the spread of invasive species. Risk Analysis, 23(6), 1141–1150. https://doi.org/10.1111/j.0272-4332.2003.00390.x

Nadifar, M., Bekker, A., Arashi, M., & Ramoelo, A. (2026). Bayesian semi-parametric spatial modeling of dispersed count data with INLA: applications in public health and climate risk. Stochastic Environmental Research and Risk Assessment, 40(2), 48.

García, J. A., Pizarro, M. M., Acero, F. J., & Parra, M. I. (2021). A Bayesian Hierarchical Spatial Copula Model: An Application to Extreme Temperatures in Extremadura (Spain). Atmosphere, 12(7), 897. https://doi.org/10.3390/atmos12070897

Ragulina, G., & Reitan, T. (2017). Generalized extreme value shape parameter and its nature for extreme precipitation using long time series and the Bayesian approach. Hydrological Sciences Journal, 62(6), 863–879.
https://doi.org/10.1080/02626667.2016.1260134

Rischmüller, A. L., et al. (2026). Bayesian hierarchical modelling of intensity-duration-frequency curves using a climate model large ensemble. Advances in Statistical Climatology, Meteorology and Oceanography, 12, 1–19. https://doi.org/10.5194/ascmo-12-1-2026

Sharkey, P., & Winter, H. C. (2017). A Bayesian spatial hierarchical model for extreme precipitation in Great Britain. arXiv. https://doi.org/10.48550/arxiv.1710.02091

Zeng, H., Huang, J., Li, Z., Yu, W., & Zhou, H. (2021). Nonstationary Bayesian Modeling of Extreme Flood Risk and Return Period Affected by Climate Variables for Xiangjiang River Basin, in South-Central China. Water, 14(1), 66. https://doi.org/10.3390/w14010066

Antoniadou, N., Stockmarr, A., Pedersen, J. W., Schmith, T., & Mikkelsen, P. S. (2025). Spatiotemporal analyses of extreme rainfall frequencies in Denmark via Bayesian hierarchical modelling using SPDE with INLA. Stochastic Environmental Research and Risk Assessment. https://doi.org/10.1007/s00477-025-03054-5

Berild, M. O., Martino, S., Gómez-Rubio, V., & Rue, H. (2022). Importance Sampling with the Integrated Nested Laplace Approximation. Journal of Computational and Graphical Statistics, 31(4), 1225–1237. https://doi.org/10.1080/10618600.2022.2067551

Gómez-Rubio, V., Cameletti, M., & Blangiardo, M. (2022). Missing data analysis and imputation via latent Gaussian Markov random fields. SORT-Statistics and Operations Research Transactions, 46(2), 217–244. https://doi.org/10.2436/20.8080.02.124

Haddad, K. (2025). A comprehensive review and application of Bayesian methods in hydrological modelling: Past, present, and future directions. Water, 17(7), 1095.

Asner, G. P., Martin, R. E., Knapp, D. E., Enquist, B. J., Helmer, E. H., Hughes, R. F., ... & Pappas, K. I. (2011). Airborne chemical spectroscopy of the Amazon basin. Proceedings of the National Academy of Sciences, 108(12), 4870–4875. https://doi.org/10.1073/pnas.1014127108

Kattenborn, T., Schiefer, F., Zarco-Tejada, P. J., & Schmidtlein, S. (2019). Advantages of SQL-based Bayesian hierarchical modelling for hyperspectral leaf area index (LAI) estimation. Remote Sensing of Environment, 229, 199–211. https://doi.org/10.1016/j.rse.2019.04.032

Verrelst, J., Camps-Valls, G., Muñoz-Marí, J., Rivera, J. P., Veroustraete, F., Clevers, J. G., & Moreno, J. (2015). Optical remote sensing and the retrieval of terrestrial vegetation bio-geophysical variables – A review. ISPRS Journal of Photogrammetry and Remote Sensing, 108, 273–290. https://doi.org/10.1016/j.isprsjprs.2015.05.005

Pilyugina, P., Chernikov, T., Smirnova, M., Zaytsev, A., Bulkin, A., Burnaev, E., ... & Anisimov, O. (2025). A physics-informed machine learning framework for permafrost stability assessment. IEEE Access.

Bakhamis, A. N., Bilal, H., Heggy, E., Al-Kuwari, M. S., & Al-Ansari, T. (2024). On the drivers, forecasts, and uncertainties of relative sea level rise in the Eastern Arabian Peninsula: A review. Regional Studies in Marine Science, 73, 103503.

Hoteit, I., Abualnaja, Y., Afzal, S., Aman, C., Antony, C., Ashok, K., ... & Zhan, P. (2025). New climate change center of Saudi Arabia: Advancing understanding and prediction for the Arabian Peninsula climate. Earth's Future, 13(12), e2025EF006296.

Green, J., Haigh, I. D., Quinn, N., Neal, J., Wahl, T., Wood, M., ... & Camus, P. (2024). A comprehensive review of coastal compound flooding literature. arXiv preprint arXiv:2404.01321.

Siunduh, E. S., Mwangi, Z., & Ikoha, A. P. (2025). Evaluating Mechanistic Data Analysis Methods for Machine Learning on Effects of Climate Change in Africa.

Koochali, A., Schichtel, P., Dengel, A., & Ahmed, S. (2022). Random noise vs. state-of-the-art probabilistic forecasting methods: A case study on crps-sum discrimination ability. Applied Sciences, 12(10), 5104.

Kopczewska, K. (2020). Applied spatial econometrics. In Applied spatial statistics and econometrics (pp. 213-287). Routledge.