Policy Research Journal
ISSN (E): 3006-7030 ISSN (P) : 3006-7022 Volume 4, Issue 4, 2026

DEVELOPMENT OF STOCHASTIC MODELS FOR ANALYZING RANDOM PROCESSES AND THEIR APPLICATIONS IN MODERN PHYSICS

Ihtisham Ulhaq[*1], Dr Raheela Bibi[2], Ameer Jan[3]

[*1]*Department of Mathematics, Sir Syed University of Engineering and Technology*
[2]*Assistant professor, Govt Nazareth college, College Education Department*
[3]*University of Makran*

[*1]ihtishamzmary@gmail.com, [2]raheelashah216@gmail.com, [3]ameerjan@uomp.edu.pk

DOI: https://doi.org/10.5281/zenodo.19875357

Keywords
stochastic processes, Brownian motion, Langevin equation, Fokker-Planck equation, stochastic thermodynamics, fluctuation theorems, open quantum systems, stochastic quantization, stochastic gravitational wave background, multiple exciton generation, non-Markovian dynamics, physics-informed neural networks

Article History
Received: 04 March 2026
Accepted: 11 April 2026
Published: 29 April 2026

Corresponding Author: *
Ihtisham Ulhaq

Abstract
Stochastic models provide a fundamental mathematical framework for analyzing random processes in modern physics, bridging microscopic fluctuations with macroscopic phenomena. This review traces the historical development from Einstein's and Langevin's foundational work on Brownian motion to contemporary applications in stochastic thermodynamics, open quantum systems, stochastic quantization (Parisi-Wu method), and the detection of stochastic gravitational wave backgrounds via pulsar timing arrays. Key mathematical tools include Fokker-Planck equations, Ito/Stratonovich interpretations of stochastic differential equations, fluctuation theorems, and unraveling techniques for density matrix evolution. These models enable the study of non-equilibrium systems, multiple exciton generation, decoherence in quantum systems, and non-Markovian dynamics. Integration with machine learning via physics-informed neural networks and emerging hardware implementations further expands their utility. Stochastic approaches are essential for understanding complex systems in statistical mechanics, quantum field theory, biophysics, and cosmology, offering predictive power where deterministic models fail.

1. Introduction

The transition of physical science from a purely deterministic framework to one fundamentally integrated with stochastic processes represents one of the most profound intellectual shifts of the modern era (Misra et al., 1979). While classical mechanics, epitomized by the Newtonian worldview, sought to describe the universe as a predictable clockwork mechanism, the exploration of the microscopic and cosmological realms has revealed a reality governed by inherent randomness and probabilistic evolution (Tsekov, 2010). The development of stochastic models mathematical structures designed to analyze systems that evolve with time according to probabilistic laws has provided the necessary tools to bridge the gap between microscopic fluctuations and macroscopic observations (Piasecki, 2006). This evolution began with the seemingly simple observation of pollen grains in water and has culminated in a multidisciplinary framework that underpins modern statistical mechanics quantum field theory non-equilibrium thermodynamics and the detection of gravitational waves (Piasecki, 2006). Figure 1 illustrates the conceptual transition from deterministic frameworks to stochastic modeling in modern physics. It highlights how stochastic processes form the foundation for diverse applications across multiple physical domains.

Figure 1: Conceptual Framework of Stochastic Processes in Modern Physics

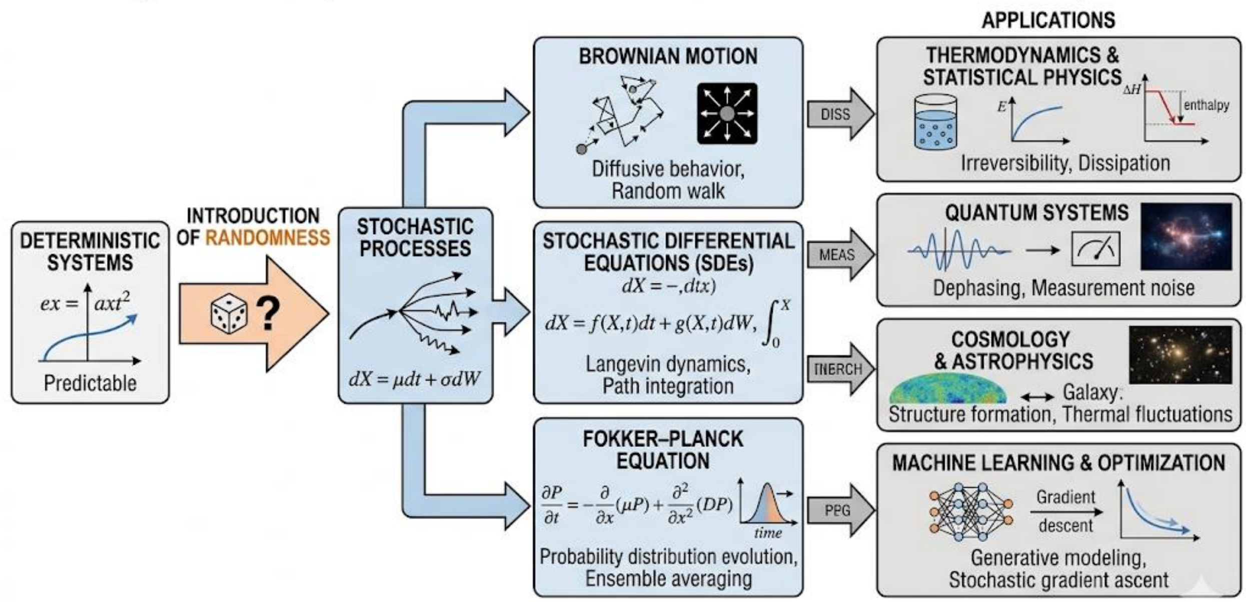


Figure 1: Conceptual Framework of Stochastic Processes in Modern Physics

2. The Genesis of Stochastic Modeling: Brownian Motion and the Atomistic Hypothesis

The historical trajectory of stochastic modeling is inextricably linked to the study of Brownian motion, a phenomenon named after the Scottish botanist Robert Brown (Bian et al., 2016). In 1827, while conducting microscopic observations of pollen grains suspended in water, Brown noted an erratic, jittery motion that never seemed to cease (Brown, 1828). Brown's systematic investigation was pivotal because he demonstrated that this motion was not a characteristic of biological life, as it was equally observable in inorganic materials, such as fragments of an Egyptian sphinx or ground-up stone (Haw, 2005). Despite the ubiquity and persistence of the phenomenon, the physics community of the nineteenth century struggled to identify its cause, often discarding various explanations such as currents within the liquid or the presence of air bubbles (Nieves et al., 2004).

The definitive explanation arrived in 1905, during Albert Einstein's "Annus Mirabilis" (Stachel, 2005). Einstein's insight was to treat the suspended particles as being in thermal equilibrium with the molecules of the surrounding fluid. He proposed that the erratic motion resulted from the collective effect of constant random bombardments by these invisible water molecules (Ermak & McCammon, 1978). By applying the principles of kinetic theory and thermodynamics, Einstein formulated a diffusion equation that related the macroscopic diffusion coefficient DD to the mean square displacement ⟨x2⟩⟨x2⟩ of the particle over time tt:

⟨x2⟩=2Dt⟨x2⟩=2Dt

This relation provided the first quantitative link between the visible movement of microscopic particles and the invisible molecular structure of matter (Tian et al., 2016). Independently, Marian von Smoluchowski and William Sutherland arrived at similar conclusions (von Smoluchowski, 1906). The experimental verification of Einstein's predictions by Jean Perrin and his students in 1908 served as the "smoking gun" that silenced skeptics of the atomic theory. Perrin's work, which utilized colloidal suspensions of gamboge to calculate Avogadro's number, earned him the Nobel Prize in Physics in 1926 (Nye, 1972).

While Einstein's approach was grounded in configuration space and probability density functions, Paul Langevin introduced an alternative mesoscopic description in 1908. Langevin's approach applied Newton's second law directly to a single Brownian particle, augmented by a "random force" representing the net molecular bombardment (Langevin, 1908). This force, now known as Gaussian white noise, is characterized by being uncorrelated in time and having a zero mean (Beyer, 2025). Langevin's formulation laid the groundwork for the modern theory of stochastic differential equations (SDEs), splitting the forces acting on a particle into a deterministic viscous drag and a stochastic fluctuation term (Renn, 2005).

Table 1. Milestones in the Historical Development of Brownian Motion and Stochastic Physics

Model Milestone	Author(s)	Key Contribution	Physical Domain
Observation (1827)	Robert Brown	Systematic study of jittery motion in organic/inorganic particles.	Botany/Physics
Diffusion Theory (1905)	Albert Einstein	Linked MSD to diffusion; proved molecular existence.	Statistical Mechanics
Kinetic Theory (1906)	Marian von Smoluchowski	Independent derivation of Brownian motion via molecular collisions.	Kinetic Theory
Langevin Equation (1908)	Paul Langevin	Introduced stochastic force; invented the mesoscopic SDE approach.	Stochastic Physics
Experimental Proof (1908)	Jean Perrin	Verified Einstein/Smoluchowski theories; calculated N_A.	Experimental Physics

3. Mathematical Formalisms: Markov Processes and Stochastic Calculus

The rigorous mathematical framework required to analyze random processes emerged from the development of probability theory and Markov process theory. A stochastic process is said to be Markovian if its future evolution depends only on its present state, rendering its past trajectory irrelevant for prediction (Annunziato & Borzi, 2018). This memoryless property enables the representation of complex dynamical systems through the Chapman–Kolmogorov equation, which governs transition probability functions across time (Sindhushree et al., 2025).

At the core of stochastic analysis lies the Fokker–Planck (FP) equation, also known as the Kolmogorov forward equation. This equation describes the temporal evolution of the probability density function (PDF) associated with a stochastic system (Sutherland, 1905). For a continuous-time stochastic differential equation (SDE) of the form:

$$dX_t = b(X_t, t)\,dt + \sigma(X_t, t)\,dW_t,$$

the corresponding Fokker–Planck equation is given by:

$$\frac{\partial f(x,t)}{\partial t} = -\sum_i \frac{\partial}{\partial x_i}[b_i(x,t)f(x,t)] + \frac{1}{2}\sum_{i,j}\frac{\partial^2}{\partial x_i\,\partial x_j}[a_{ij}(x,t)f(x,t)],$$

where $a_{ij}(x,t)$ represents the diffusion tensor, typically defined as $a = \sigma\sigma^T$. This formulation simultaneously captures deterministic drift and stochastic diffusion effects (Perrin, 1909). The FP framework has become fundamental across multiple disciplines, including statistical physics and quantitative finance, where models such as geometric Brownian motion underpin option pricing theory (Beyer, 2025).

A key refinement in stochastic modeling is the distinction between Itô and Stratonovich interpretations of stochastic integrals. The Itô formulation evaluates integrands at the beginning of each time increment and is widely used in financial mathematics due to its martingale-preserving properties (Lemons & Gythiel, 1997). In contrast, the Stratonovich interpretation evaluates integrands at midpoints and is often preferred in physical systems because it preserves classical chain-rule calculus and better approximates noise with finite correlation time (Donado et al., 2017). Recent advances have introduced quantum noise homogenization techniques, which aim to reconcile these frameworks by coarse-graining non-Markovian dynamics into effective white-noise representations (Mukherjee, 2026).

4. Stochastic Thermodynamics: Entropy and Work at the Nanoscale

The extension of thermodynamic principles to systems dominated by thermal fluctuations has given rise to the field of stochastic thermodynamics. This framework enables the definition of work, heat, and entropy production along individual stochastic trajectories within well-defined non-equilibrium ensembles (Seifert, 2012). In contrast to classical thermodynamics, which is restricted to macroscopic equilibrium systems, stochastic thermodynamics is applicable to microscopic systems such as colloidal particles in optical traps, molecular motors, and enzymatic reactions, where thermal noise plays a dominant role. In such regimes, transient violations of the second law of thermodynamics may occur over short time intervals (Dillard, 2021).

The apparent paradox of second-law violations is resolved through fluctuation theorems (FTs), which provide exact statistical relations governing the probability distributions of thermodynamic quantities in non-equilibrium systems (Moxley, 2007). In particular, the steady-state fluctuation theorem (SSFT) extends the second law by demonstrating that entropy-reducing trajectories are not forbidden, but instead become exponentially unlikely as either the observation time or system size increases. Among the most significant results in this field is the Jarzynski equality, which relates the equilibrium free energy difference ΔF between two thermodynamic states to the exponential average of the non-equilibrium work W performed during a transformation (Jarzynski, 1997):

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

where $\beta = (k_B T)^{-1}$ is the inverse thermal energy scale. This identity provides a powerful bridge between equilibrium and non-equilibrium thermodynamics, allowing equilibrium free energy differences to be computed from irreversible processes. As a result, it has had a transformative impact on single-molecule experimental techniques in biophysics and molecular-scale measurements (Pyurbeeva et al., 2023).

Table 2. Key Thermodynamic Relations for Trajectory-Based Statistical Mechanics

Thermodynamic Relation	Context	Physical Significance
Jarzynski Equality	Transitions between equilibrium states.	Relates non-equilibrium work to equilibrium free energy.
Crooks Theorem	Forward and time-reversed processes.	Quantifies the irreversibility of a process.

Hatano-Sasa Relation	Transitions between steady states.	Generalizes the Clausius inequality for NESS.
Fluctuation-Dissipation	Near-equilibrium linear response.	Links spontaneous fluctuations to system response.

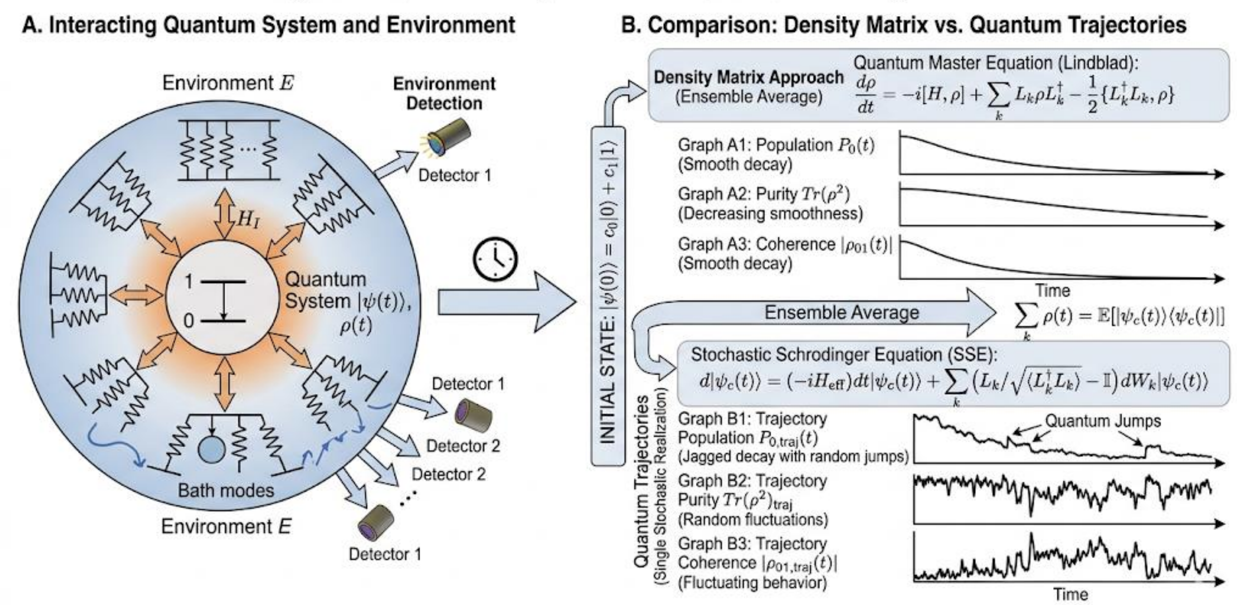
5. Open Quantum Systems: Dissipation, Decoherence, and Stochastic Trajectories

In the quantum regime, systems continuously interact with their surrounding environment, resulting in the loss of quantum coherence, a phenomenon known as decoherence. The standard mathematical framework for describing such dissipative dynamics is the Lindblad master equation, which governs the time evolution of the system's density matrix $\rho(t)$. However, the numerical simulation of large-scale open quantum systems is computationally challenging, as the number of parameters in the density matrix scales exponentially as $O(d^{2L})$, where $d$is the local Hilbert space dimension and $L$is the number of subsystems (Adhikari & Baer, 2024).

To address this "curse of dimensionality," stochastic unravelling techniques have been developed, such as the Monte Carlo wave function (MCWF) method. These approaches replace the full density matrix evolution with an ensemble of individual quantum trajectories, where pure states evolve deterministically between stochastic "quantum jump" events (Triana & Herrera, 2022). The ensemble average over these trajectories reconstructs the full density matrix dynamics.

Recent methodological advances include the tensor jump method (TJM), which integrates quantum jump unravellings with matrix product state representations, enabling efficient simulation of large-scale many-body systems, including spin networks with up to a thousand spins (Sander et al., 2025). Figure 2 illustrates this stochastic trajectory framework for open quantum systems, highlighting how ensembles of quantum jumps approximate the full density matrix evolution.

Figure 2: Quantum Trajectories in Open Quantum Systems



6. Stochastic Quantization: The Parisi-Wu Method

A powerful application of random processes in theoretical physics is stochastic quantization, originally proposed by Parisi and Wu in 1981. This framework provides an alternative approach to quantization in which a d-dimensional quantum field theory is reformulated as the equilibrium limit of a $(d+1)$-dimensional classical stochastic process, where the additional dimension is an artificial “stochastic time” τ(Zhang et al., 2025). In this formulation, the field evolves along τaccording to a Langevin-type equation driven by Gaussian white noise:

$$\frac{\partial\Phi(x,\tau)}{\partial\tau} = -\frac{\delta S[\Phi]}{\delta\Phi(x,\tau)} + \xi(x,\tau),$$

where $S[\Phi]$denotes the Euclidean action of the field theory (Gozzi, 1983), and $\xi(x,\tau)$represents a stochastic noise term with specified correlation properties.

Stochastic quantization is particularly advantageous in gauge field theories, as it enables quantization without the need for explicit gauge fixing. Consequently, it circumvents the introduction of Faddeev–Popov ghost fields and associated complications that arise in conventional quantization procedures (Parisi & Wu, 1981).

7. Astrophysics and Cosmology: The Stochastic Gravitational Wave Background

Stochastic models are also applied in the observation of the largest structures in the universe. While ground-based interferometers detect discrete signals, the universe is filled with a "background" of radiation known as the Stochastic Gravitational Wave Background (SGWB). The SGWB is the superposition of a vast number of unresolved signals from distant binary populations and primordial sources (Agazie et al., 2023).

In 2023, Pulsar Timing Array (PTA) collaborations announced the detection of a nanohertz-frequency SGWB. By measuring arrival times of radio pulses over decades, researchers identified spatial correlations matching the Hellings-Downs curve. Stochastic modeling is essential for disentangling these potential sources, such as scalar-induced gravitational waves (SIGWs) and primordial black holes (PBHs) (Mølmer et al., 1993).

Table 3. Sources and Frequency Characteristics of the Stochastic Gravitational Wave Background

GW Source Type	Frequency Range	Origin/Mechanism	Detection Method
Stellar-Mass Binaries	10 Hz – 10 kHz	Mergers of BHs and neutron stars.	Ground-based (LIGO/Virgo)
Supermassive BH Binaries	1 nHz – 100 nHz	Merging galaxies; dynamical friction.	Pulsar Timing Arrays (PTAs)
Primordial Background	Broadband	Inflation, cosmic strings, phase transitions.	PTAs, LISA, CMB Distortions
Galactic Binaries	0.1 mHz – 100 mHz	White dwarf and stellar-mass BH pairs.	Space-based (LISA)

8. Non-Equilibrium Phase Transitions and Criticality

The study of non-equilibrium systems has revealed collective phenomena qualitatively different from thermal equilibrium. Transitions often occur between fluctuating active phases and absorbing states from which the system cannot escape once reached. Models like the Asymmetric Simple Exclusion Process (ASEP) and directed percolation are used to analyze these transitions (Hinrichsen, 2000). In these systems, stochasticity can act as a mechanism for self-organization, and operating near the "tipping point" of these transitions may offer functional advantages (Sindhushree et al., 2025).

9. The Convergence of Stochastic Physics and Machine Learning

The intersection of stochastic modeling and machine learning (ML) has emerged as a transformative frontier. Physics-Informed Neural Networks (PINNs) have become a key tool for solving inverse problems by embedding physical laws such as Fokker-Planck equations directly into the loss function (Antonion et al., 2024). High-dimensional system identification frameworks are being developed to recover both deterministic drift and noise structures directly from trajectory data (Zhong et al., 2026).

10. Future Directions: Non-Markovianity and Neuromorphic Hardware

The future of stochastic modeling lies in the transition from Markovian to non-Markovian frameworks. Many realistic physical systems exhibit history-dependent evolution, and in open quantum systems, non-Markovian memory effects manifest as "information backflow" (Siltanen, 2022). Furthermore, physical principles of stochasticity are being integrated into computing hardware; oxygen-gradient memristors transform inherent conductance drift into a hardware learning-rate clock that facilitates stable continual reinforcement teaching (Carter, 2026).

11. Synthesis and Final Perspectives

The development of stochastic models has fundamentally redefined the methodology of modern physics. From the validation of the atomistic hypothesis to the detection of primordial gravitational waves, the ability to analyze random processes has proven key to understanding complex systems (Einstein, 1905). As the field integrates further with artificial intelligence, the role of stochasticity is shifting from a source of error to a fundamental mechanism for learning and self-organization (Raissi et al., 2019).

Conclusion

The development of stochastic models has fundamentally transformed our understanding of physical reality by incorporating inherent randomness and probabilistic evolution into the core of theoretical and experimental physics. From validating the atomic hypothesis through Brownian motion to enabling the detection of primordial gravitational waves and simulating large-scale open quantum systems, these models have proven indispensable across scales from nanoscale molecular motors to cosmological phenomena. Advances in stochastic thermodynamics, quantum trajectory methods, and integration with machine learning are opening new frontiers in non-equilibrium physics, quantum information, and complex system modeling. Future progress will likely focus on efficient simulation of non-Markovian and high-dimensional systems, hardware-level stochastic computing, and deeper interdisciplinary applications in quantum biology and climate modeling. As physics continues to embrace complexity and uncertainty, stochastic frameworks will remain central to both fundamental discovery and technological innovation in the 21st century.

12. REFERENCES

Adhikari, S., & Baer, R. (2024). Stochastically bundled dissipators for the quantum master equation. arXiv. https://doi.org/10.1021/acs.jctc.5c00145

Agazie, A., Anumarlapudi, A., Archibald, A. M., Arzoumanian, Z., Baker, P. T., Bécsy, B.,... & Witt, C. A. (2023). The NANOGrav 15 yr data set: Evidence for a stochastic gravitational-wave background. *The Astrophysical Journal Letters*, *951*(1), L8. https://doi.org/10.3847/2041-8213/acdac6

Annunziato, M., & Borzì, A. (2018). A Fokker-Planck control framework for stochastic systems. *EMS Surveys in Mathematical Sciences*, *5*(1/2), 65–98.(https://doi.org/10.4171/EMSS/27)

Antonion, K., Wang, X., Raissi, M., & Joshie, L. (2024). Machine learning through physics–informed neural networks: Progress and challenges. *Academic Journal of Science and Technology*, *9*(1), 46–49. https://doi.org/10.54097/b1d21816

Beyer, K. (2025). A comparative analysis of stochastic models for stock price. *QFE*, *3*(2), 211–229. https://doi.org/10.3934/QFE.2025021

Bian, X., Kim, C., & Karniadakis, G. E. (2016). 111 years of Brownian motion. Soft Matter, 12(30), 6331–6346. https://doi.org/10.1039/c6sm01153e

Brown, R. (1828). A brief account of microscopical observations made in the months of June, July and August 1827, on the particles contained in the pollen of plants; and on the general existence of active molecules in organic and inorganic bodies. Philosophical Magazine, 4(21), 161–173.

Carter, A. (2026, April 10). Oxygen-gradient memristor turns slow conductance drift into a precise hardware learning-rate clock. *Intelligent Living*. https://www.intelligentliving.co/oxygen-gradient-memristor-learning/

Dillard, J. (2021). Nonlocality in a Stochastic Approach to Quantum Mechanics and Quantum Field Theory. William & Mary.

Donado, F., Moctezuma, R. E., López-Flores, L., Medina-Noyola, M., & Arauz-Lara, J. L. (2017). Brownian motion in non-equilibrium systems and the Ornstein-Uhlenbeck stochastic process. Scientific Reports, 7. https://doi.org/10.1038/s41598-017-12737-1

Einstein, A. (1905). Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen [On the movement of small particles suspended in stationary liquids required by the molecular-kinetic theory of heat]. *Annalen der Physik*, *322*(8), 549–560. https://doi.org/10.1002/andp.19053220806

Ermak, D. L., & McCammon, J. A. (1978). Brownian dynamics with hydrodynamic interactions. The Journal of Chemical Physics, 69(4), 1352–1360. https://doi.org/10.1063/1.436761

Gozzi, E. (1983). Functional-integral approach to Parisi-Wu stochastic quantization: Scalar theory. *Physical Review D*, *28*(8), 1922–1930.(https://doi.org/10.1103/PhysRevD.28.1922)

Haw, M. D. (2005). Luke Howard and Robert Brown: History and science of clouds and molecules. Weather, 60(11), 335–339.

Hinrichsen, H. (2000). Nonequilibrium critical phenomena and phase transitions into absorbing states. *Advances in Physics*, *49*(7), 815–958. https://doi.org/10.1080/00018730050198152

Jarzynski, C. (1997). Nonequilibrium equality for free energy differences. *Physical Review Letters*, *78*(14), 2690–2693.(https://doi.org/10.1103/PhysRevLett.78.2690)

Langevin, P. (1908). Sur la théorie du mouvement brownien. *Comptes Rendus de l'Académie des Sciences (Paris)*, *146*, 530–533.

Lemons, D. S., & Gythiel, A. (1997). Paul Langevin's 1908 paper "On the Theory of Brownian Motion". *American Journal of Physics*, *65*(11), 1079–1081. https://doi.org/10.1119/1.18725

Misra, B., Prigogine, I., & Courbage, M. (1979). From deterministic dynamics to probabilistic descriptions. Proceedings of the National Academy of Sciences, 76(8), 3607-3611. https://doi.org/10.1073/pnas.76.8.3607

Mølmer, K., Castin, Y., & Dalibard, J. (1993). Monte Carlo wave-function method in quantum optics. Journal of the Optical Society of America B, 10(3), 524–538. https://doi.org/10.1364/josab.10.000524

Moxley, R. A. (2007). Ultimate realities: Deterministic and evolutionary. The Behavior Analyst, 30, 59-77. https://doi.org/10.1007/bf03392146

Mukherjee, A. (2026). A resolution of the Ito-Stratonovich debate in quantum stochastic processes. *Modern Mathematics and Physics*, 2(1), 3. https://doi.org/10.3390/mmphys2010003

Nieves, J. L., Romero-Rochín, V., & Sahagun, A. (2004). Brownian motion and the emergence of the concept of probability in physics. Revista Mexicana de Física, 50(1), 84–91.

Nye, M. J. (1972). Molecular reality: A case study of the scientific career of Jean Perrin. MacDonald.

Parisi, G., & Wu, Y. S. (1981). Perturbation theory without gauge fixing. *Scientia Sinica*, *24*(4), 483–496.

Perrin, J. (1909). Brownian movement and molecular reality. Annales de Chimie et de Physique, 18, 1–114.

Piasecki, J. (2006). Centenary of Marian Smoluchowski's theory of Brownian motion. University of Connecticut.

Pyurbeeva, E., Thomas, J. O., & Mol, J. A. (2023). Non-equilibrium thermodynamics in a single-molecule quantum system. Materials for Quantum Technology, 3(2), 025003.

Raissi, M., Perdikaris, P., & Karniadakis, G. E. (2019). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, *378*, 686–707. https://doi.org/10.1016/j.jcp.2018.10.045

Renn, J. (2005). Einstein's invention of Brownian motion. *Annalen der Physik*, *14*(S1), 23–37. https://doi.org/10.1002/andp.200410131

Sander, A., Fröhlich, M., Eigel, M., Eisert, J., Gelß, P., Hintermüller, M., Milbradt, R. M., Wille, R., & Mendl, C. B. (2025). Large-scale stochastic simulation of open quantum systems. arXiv. https://doi.org/10.48550/arxiv.2501.17913

Seifert, U. (2012). Stochastic thermodynamics, fluctuation theorems and molecular machines. *Reports on Progress in Physics*, *75*(12), 126001. https://doi.org/10.1088/0034-4885/75/12/126001

Siltanen, S. (2022, September 26). *Non-Markovian memory effects in quantum systems*. Emergent Mind. https://www.emergentmind.com/topics/non-markovian-memory-effects

Sindhushree, K., Sharma, A., Patel, R., & Mehta, S. (Eds.). (2025). Theory and applications of stochastic processes. IntechOpen. https://doi.org/10.5772/intechopen.1001

Stachel, J. (2005). Einstein's miraculous year: Five papers that changed the face of physics. Princeton University Press.

Sutherland, W. (1905). A dynamical theory of diffusion for non-electrolytes and the molecular mass of albumin. Philosophical Magazine, 9(54), 781–785.

Tian, L., Ahmadi, G., & Tu, J. (2016). Brownian diffusion of fibers. Aerosol Science and Technology, 50(5), 474–486. https://doi.org/10.1080/02786826.2016.1165340

Triana, J. F., & Herrera, F. (2022). Open quantum dynamics of strongly coupled oscillators with multi-configuration time-dependent Hartree propagation and Markovian quantum jumps. The Journal of Chemical Physics, 157(19). https://doi.org/10.1063/5.0119293

Tsekov, R. (2010). Brownian motion of molecules: the classical theory. Ann. Univ. Sofia, Fac. Chem., 88(1), 57-66. https://doi.org/10.48550/arxiv.1005.1490

von Smoluchowski, M. (1906). Zur kinetischen Theorie der Brownschen Molekularbewegung und der Suspensionen [On the kinetic theory of Brownian molecular motion and suspensions]. Annalen der Physik, 326(14), 756–780.

Zhang, L., Wang, Y., Chen, X., Li, J., & Liu, H. (2025). Large-scale stochastic simulation of open quantum systems. Physical Review Research, 7(1), Article 013141.

Zhong, M., Cialenco, I., & Guo, Z. (2026). Noise-aware system identification for high-dimensional stochastic dynamics. *OpenReview*.(https://openreview.net/forum?id=B2jdDDq7GF)