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TWO-DIMENSIONAL RIGID IMAGE REGISTRATION USING THE COARSE SEARCH APPROCH

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Abstract
The process or technique of matching the appearance of two or more images by determining an alignment between them is known as image registration. This is basically aligns two images geometrically. In this study, two-dimensional image registration is presented using the rigid group. This group is a finite dimensional group under composition; it is four-dimensional in this particular case. The dimensions of the rigid group include scaling, rotation, and translations along the axes. This paper presents a method that uses a discretized objective function to create rigid transformations. Based on SSD (sum of the squares of the distances between the pixel intensities), this objective function calculates the difference between the images. The coarse search approach is used for the optimization. The proposed algorithm is implemented on a variety of images, mostly our own captured images. The numerical examples illustrate the effectiveness of the proposed algorithm.

1. INTRODUCTION

The process or technique of matching the appearance of two or more images by determining an alignment between them is known as image registration. This essentially aligns two images geometrically [1-3]. It has been extensively researched and applied in a number of fields, such as medical imaging [4] and biology (in the context of morphometrics) [5]. The collection of acceptable transformations that are used to deform the images is one of the fundamental image registration choices [6]. In this paper, we use the rigid group for two-dimensional picture registration. This paper focuses on the use of a coarse search Algorithm for objective function optimization. The proposed method is applied to several examples in order to demonstrate its effectiveness in image registration.

1.1 Objective Function

Consider the couple of images I_1 and I_2, usually referred to as source and target, are defined in a particular domain μ (typically, $\mu \subset \mathbb{R}^2$), for 2D grey scale images), image registration can be described as a minimizing objective or error function that evaluates the discrepancy between pictures with a deformation variation of φ^{-1} which implement on any of the images. (Use of φ^{-1} is clarified in [6]). One of the objective functions is:

$$E_{I_1,I_2}(\varphi) = \int_{\mu} \|I_1 \circ \varphi^{-1}(x_1,x_2) - I_2(x_1,x_2)\|^2 \, dx_1 dx_2, \qquad \forall (x_1,x_2) \in \mu \tag{1.1}$$

After applying the transformation φ and the optimization approach, the norm $\|\cdot\|$ is used to calculate the discrepancy between the target picture and the transformed source. The above equation (1.1) is used to register two-dimensional greyscale pictures. However in this paper, we consider the rigid group, which is formed by translation along the x-axis and y-axis, rotations, and scalings, consisting of four parameters in two dimensions. This allows for a global transformation of space.

2. REGISTRATION OF IMAGES WITH RIGID REGISTRATION

In image registration, One of the most often utilized group is the rigid group [6]. For this group the major indicators are scalings, rotations, and arbitrary translations (in 2-dimensional) rigid group [6-7]. The transformation (which is now called the rigid transformation) associated with rigid group is explained below:

2.1 Rotation

Suppose an arbitrary $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$. Let $\theta \in [0, 2\pi)$ be the angle of rotation transformation φ_1 to generate the new point $\begin{pmatrix} x' \\ y' \end{pmatrix} \in \mathbb{R}^2$ as defined by:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \varphi_1\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

2.2 Scaling

Suppose an arbitrary $\lambda \in \mathbb{R}^2$ is a non-zero number so uniform scaling transformation φ_2 over $\begin{pmatrix} x \\ y \end{pmatrix}$ is defined below:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \varphi_2\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

2.3 Translation

Suppose $t_x, t_y \in \mathbb{R}$ are the translations in the horizontal and vertical axis respectively. The translation transformation φ_3 over $\begin{pmatrix} x \\ y \end{pmatrix}$ is defined below:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \varphi_3\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x + t_x \\ y + t_y \end{pmatrix}$$

The rigid transformation is the combination of the all above mentioned transformations. The compact form of the rigid transformation φ is defined below:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \varphi \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= (\varphi_3 \circ \varphi_2 \circ \varphi_1)\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$= \lambda \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}, \quad \theta \in [0, 2\pi), 0 \neq \lambda \in \mathbb{R} \qquad (2.1)$$

The collection of all such φ forms a group under composition, which is known as rigid

Group. (From Equation 2.1) The inverse of a rigid transformation is:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}^{-1} \left\{ \begin{pmatrix} x' \\ y' \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \end{pmatrix} \right\}$$

$$= \frac{1}{\lambda} \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}^{-1} \left\{ \begin{pmatrix} x' \\ y' \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \end{pmatrix} \right\} \qquad (2.2)$$

Since φ and φ^{-1} are both diffeomorphisms, the rigid group is a subgroup of all diffeomorphisms of $\mathbb{R}^2$.

Definition 2.1. Let's suppose that I_1 *and* I_2 are two greyscale images. We define a matching functional:

$$E(\varphi) = \int_{\mu} (I_1 \circ \varphi^{-1}(x) - I_2(x))^2 dx\, dy, \qquad x = (x, y)^T \in \mu \subset \mathbb{R}^2 \qquad (2.3)$$

If φ denotes a rigid group element, then the technique of determining a φ that minimises Equation (2.3) is known as rigid registration.

The continuous form of the objective function is Equation (2.3). We must discretise this continuous form in order to perform numerical computations. Therefore, we must specify a discrete domain, which we denote as K, which is basically a domain of discrete pictures. Suppose $L \in Z^+$ and let,

$$K = \{ ([0, L-1]/(L-1) - 0.5) \times ([0, L-1]/(L-1) - 0.5) \}$$

Is a grid of squares with L^2 grid nodes. Figure (2.1) shows an example with $L = 100$ grid nodes. We shall utilise $L = 100$ all over this thesis, unless otherwise specified.

Discrete objective function is defined as follows (in which x_{ij} indicate the picture component at location (i, j) of an image matrix):

$$E(\varphi) = \sum_{i=1}^{L} \sum_{j=1}^{L} \{(I_1 \circ \varphi^{-1})(x_{ij}) - I_2\}^2 \qquad (2.4)$$

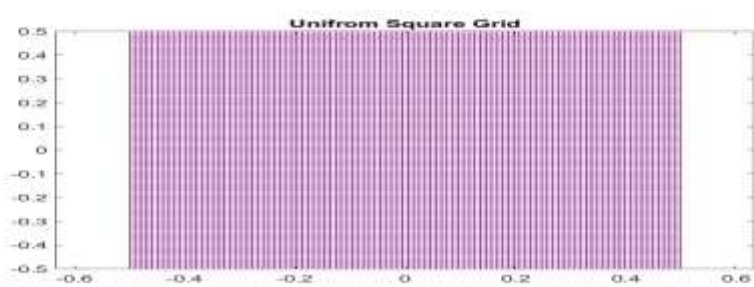


Figure 2.1: The grid domain has 10,000 node points in [-0.5, 0.5] × [-0.5, 0.5] range along with L = 100 (there are 100 node points on each side).

The calculation for $I_1 \circ \varphi^{-1}(x_{ij})$ are as follows:

1. If $\varphi^{-1}(x_{ij}) \in \mu$, we apply bilinear interpolation.
2. We utilise a constant background value if $\varphi^{-1}(x_{ij}) \notin \mu$.

The images in the registration examples in this paper are always assumed to have constant backgrounds of 0 (black) or 1 (white), although in some cases the image does not reach its background value entirely before the boundaries of the image are reached.

Our goal is to minimize $E(\varphi)$ for $\varphi \in G$. Equation (2.4) represents the general form of the least-squares optimisation function for any choice of planar transformation group (that is, G is a Lie group that acts on the plane; in this paper, G will be a rigid group). This is a numerical optimizations problem [9]. Optimization is a vast field with numerous known algorithms, the application of which is determined by the nature of the objective function. Our goal here is not to survey the topic, but rather to show some easy applications. In this paper, we focus on a coarse search method for solving the optimization.

3. COARSE SEARCH METHOD

In the light of recent optimisation studies, it may appear that it is not worth bothering with. In reality, it is still worth thinking about. First, calculating the derivative is optional, i.e., the function φ does not necessarily have to be differentiable. Second, and perhaps more importantly, the function φ may have several local minima dispersed over the domain and even nested in a fractal-like manner. This can make finding the global minimum problematic for local, derivative-based approaches like gradient descent [8]. Coarse search at least surveys the entire domain [8], or a large part of it, and can provide a solid beginning guess or guesses for more complex methods.

The coarse search Algorithm [6] to minimize the Equation (2.4):

Input: I_1 and I_2: source and target images

Output: War φ^{-1} and deformed image $I_1 \circ \varphi^{-1}$

for $\theta = -\pi : \pi/9 : \pi$ **do**
  for $\lambda = 0.1 : 0.1 : 1.5$ **do**
    for $t_x = -0.5 : 0.1 : 0.5$ **do**
      for $y_x = -0.5 : 0.1 : 0.5$ **do**
        use bi-linear interpolation to compute transformed version of the source, $I_1 \circ \varphi^{-1}(x_{ij}) \forall x_{ij} \in S$ computes
        $d = \| I_1 \circ \tilde{\varphi}^{-1} I_2(x_{ij} \|^2 \; \forall x_{ij} \in S$

for the minimum value of d, computes $\varphi^{-1} = \tilde{\varphi}^{-1}$

compute $I_1 \circ \varphi^{-1}$

In the coarse search method the objective function is evaluated at many points $\varphi \in G$; coarse search Algorithm presents an overview of the method, which consists of four nested loops. A decision needs to be made as to how many points φ to check, and how they should be distributed in G. The rigid group is 4-dimensional for registration of two dimensional images, and the coarse search can take φ to lie in a grid of values of θ, λ, t_x and t_y coarse search Algorithm. The number of values computed in the algorithm are: M_θ = 19, M_λ = 15, M_x = 11, M_y = 11, Hence the search is over the product of these sizes, requiring a total of 34,485 evaluations of the objective function. Thus, although the coarse search method is extremely simple, it is necessary to search over relatively small numbers of values for each parameter or else the computational cost becomes prohibitive.

4. RIGID REGISTRATION THROUGH COARSE SEARCH ALGORITHM

In this section, we present the application of the coarse search Algorithm on real-world data. The source images are transformed using known rigid transformations to create the synthetic target images. Three examples are examined.

Example 4.1: For this example, we consider a pair of non-smooth own captured images illustrated in Figure (4.1). To generate the target image, a rigid transformation is applied to the source image. The transformation parameters used in this example are: $\theta = \pi/3,\ \lambda = 1.2,\ t_x = -0.2,\ t_y = 0.2$ coarse search Algorithm is then applied. The optimized parameters obtained at the end of the optimizations process are: $\theta_{opt} = \pi/3,\ \lambda_{opt} = 1.2,\ t_{x_{opt}} = -0.2,\ t_{y_{opt}} = 0.2.$ These optimised parameters are then used to transform the source image. Figure (4.2) shows the image registration results. A perfect registration is obtained.

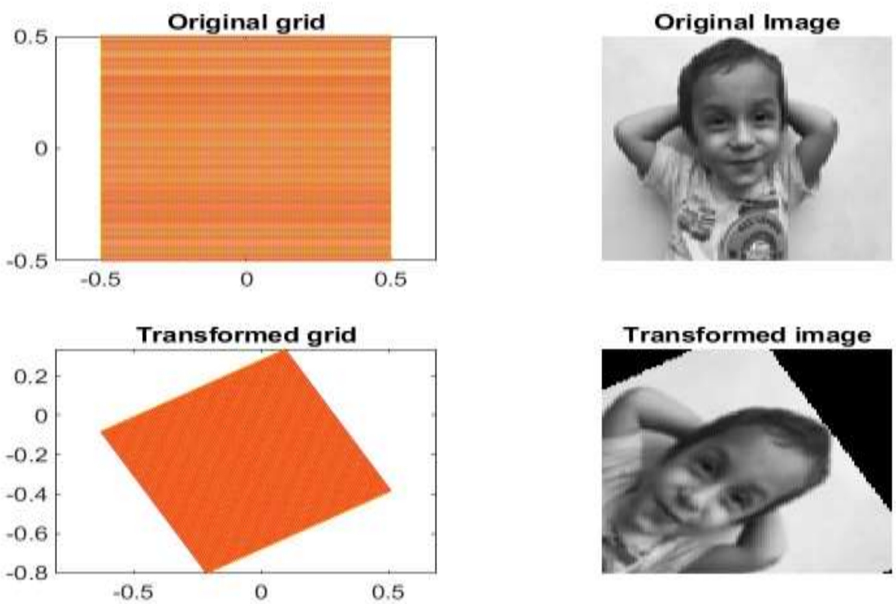
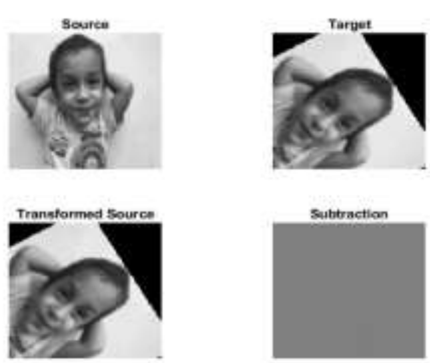
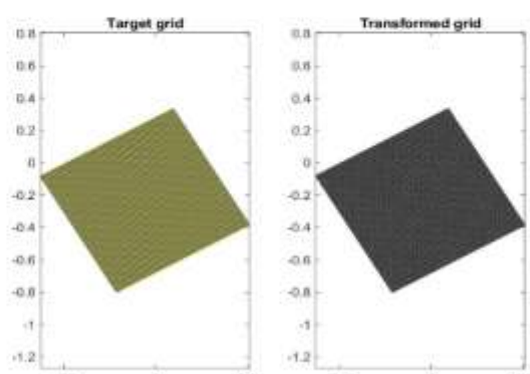


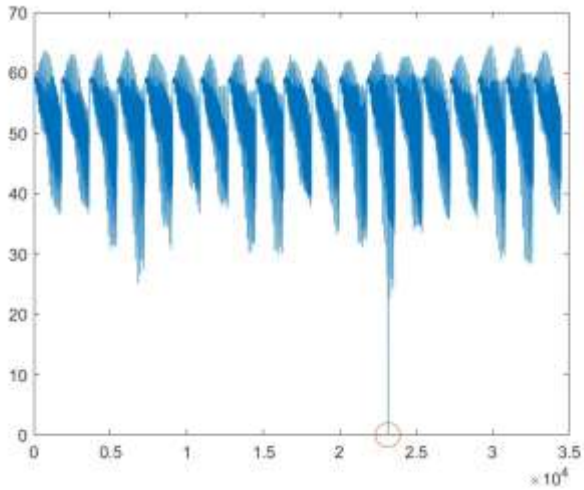
Figure 4.1: In Ex: (4.1), the non-smooth images have been used. The transformed image is generated from the original image using rigid transformation.



(a)



(b)

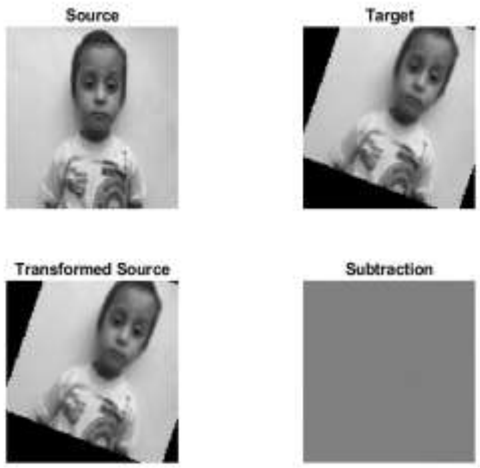


(c)

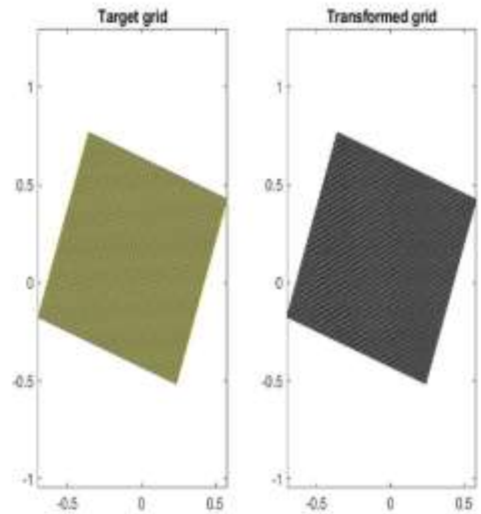
Figure 4.2: (a): A perfect registration is obtained as a result of rigid registration along with coarse search for Ex: (4.1). (b): Contains corresponding grid plots of the transformed source and target. (c): All objective function values for Ex: (4.1).

Example 4.2: Another non-smooth picture pair can be seen in Figure (4.3). Where the target is obtained from the source image using a rigid transformation with: $\theta = \frac{\pi}{9},\ \lambda = 1,\ t_x = 0.1,\ t_y = -0.1$ parameters for this example. We will now apply coarse search Algorithm. The optimized parameters obtained at the end of the optimization process are: $\theta_{opt} = \frac{\pi}{9},\ \lambda_{opt} = 1,\ t_{x_{opt}} = 0.1,\ t_{y_{opt}} = -0.1$. The optimised parameters are then used to transform the source. A perfect registration is accomplished yet again. The image Registration results are shown in Figure (4.4).

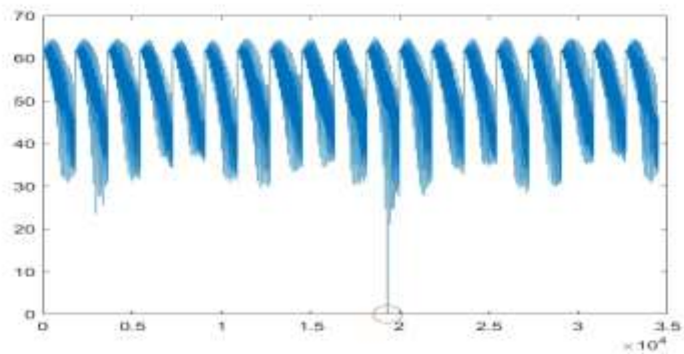
Figure 4.3: In Ex: (4.2), the non-smooth images have been used. The target picture is produced by employing rigid transformation to original image.



(a)



(b)

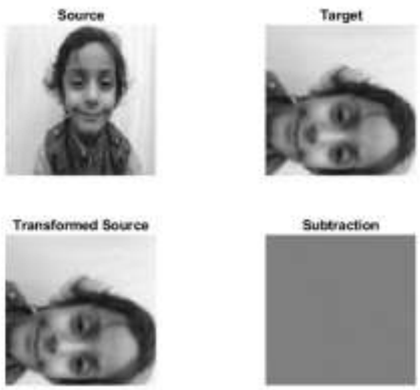


(c)

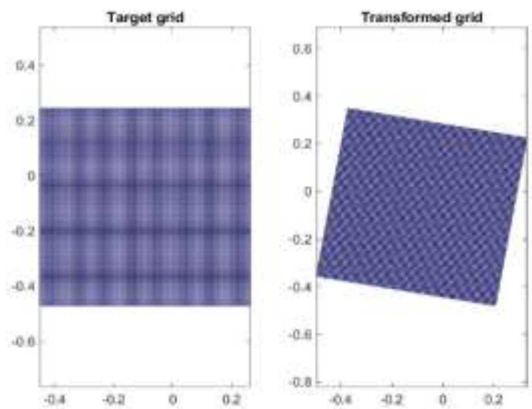
Figure 4.4: (a) The result is a perfect registration. For Ex: (4.2), Rigid registration with coarse search Algorithm. (b): Contains corresponding grid plots of the transformed source and target. (c): All objective function values for Ex: (4.2)

Example 4.3: In this example, we will look at some other set of non-smooth pictures where the target is obtained from the source image through a rigid transformation shown in Figure (4.5) The parameters for this transformation are as follows: $\theta = \frac{\pi}{2}(1.5708)$, $\lambda = 1.4$, $t_x = -0.15$, $t_y = 0.13$. The optimised parameters are returned by coarse search Algorithm that are: $\theta_{opt} = 1.7453$, $\lambda_{opt} = 1.4$, $t_{x_{opt}} = -0.1$, $t_{y_{opt}} = 0.1$. Although these are distinct from those that were use to build the target. The issue would be that the range of values allowed for coarse search does not contain the actual values that were used to build the target. Figure (4.6) shows the image registration results, which show acceptable but not perfect registration.

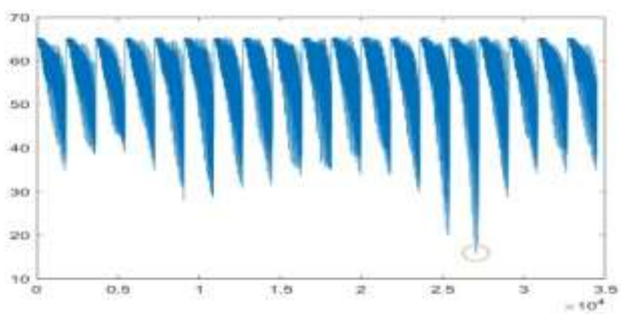
Figure 4.5: The non-smooth images have been used for this Ex (4.3). The target picture is produced by employing rigid transformation to source image.



(a)



(b)



(c)

Figure 4.6: (a): Result of rigid registration by coarse search in above example. A good but imperfect registration has been received. (b): Contain grid plots of the transformed source and target pictures (c): All objective function values. It's worth noting that the y-axis values just go downward to 15.9289, rather than 0 just like in prior cases.

5. CONCLUSION

According to the experimental results, the coarse search Algorithm works well in selected examples and a perfect registration was obtained for two examples (such as Examples (**4.1** and **4.2**). But the coarse search Algorithm also has some shortcoming. Specifically, in certain situations (such as Example **4.3**), the algorithm is unable to achieve a perfect registration, which could be explained by the insufficient resolution of the search grid or its propensity to be affected by local minima. These findings suggest that although the coarse search approach is straightforward and efficient in some situations, it is not always reliable for all types of image registration problems. However, coarse search has the advantage of surveying the entire domain and providing a good initial guess for more complex optimization methods. Therefore, coarse search can be effectively used as an initialization step for more advanced optimization techniques.

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